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enquiringminds.org
info@enquiringminds.org
+61 448 218652

Specialist Maths Units 3/4

Circular Functions

Practice Questions

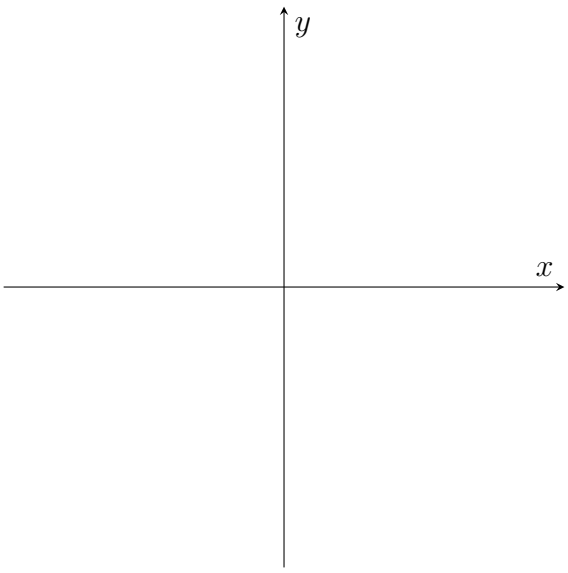
Short Answer Questions

Question 1

Consider the function $f(x) = \dfrac{\pi}{4} + 2\sin^{-1}\left(\dfrac{x-1}{2}\right)$

a) State the maximal domain and range of $f(x)$.

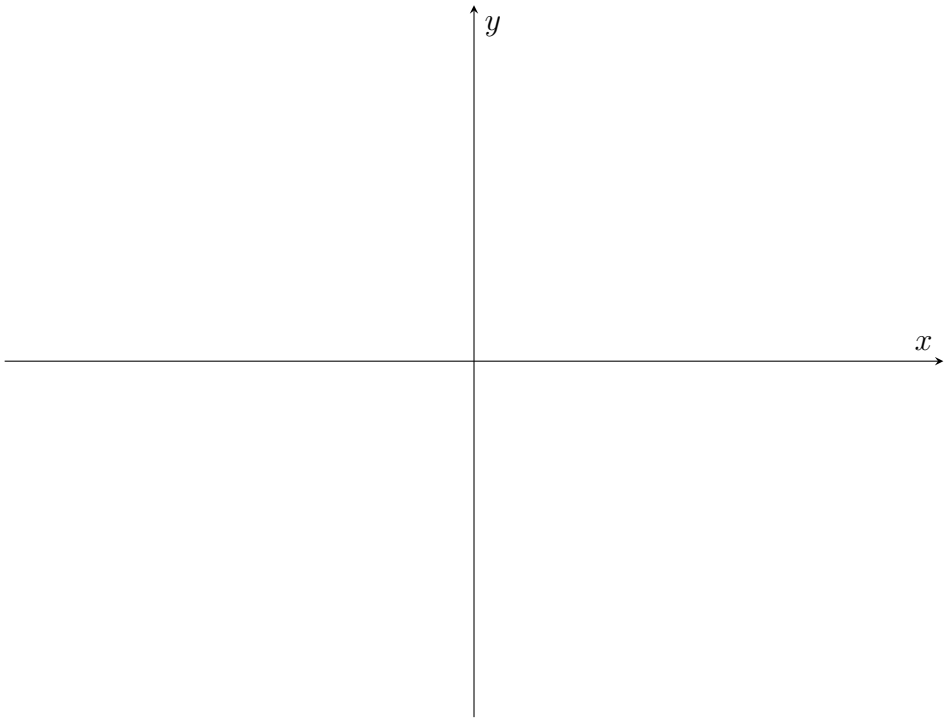
b) Sketch the graph of $y = f(x)$ on the axes below. Label any key features with their coordinates.



c) Find the solution to $f(x) = -\dfrac{\pi}{12}$

Question 2

Sketch the graph of the function $f(x) = \sec\left(x - \frac{\pi}{4}\right)$ over $[-\pi, \pi]$. Write the equations of all asymptotes and label the exact coordinates of turning points, intercepts and end points.



Question 3

If $\sin(x) = \frac{1}{2}$ and $x \in \left[\frac{\pi}{2}, \pi\right]$, then find the exact value of $\sin\left(\frac{x}{2}\right)$.

Question 4

a) Find and exact value for $\cos^{-1}\left(\cos\left(-\frac{\pi}{4}\right)\right)$

b) Find and exact value for $\tan\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$

Question 5

Use double angle formulas to find the exact value of $\cos\left(\frac{\pi}{8}\right)$

Question 6

State the exact implied domain and range of the function $f(x) = \cos^{-1}(3x - 1) + 2$

Question 7

Solve the following equations over $[-\pi, \pi]$

a) $3\cos(2x) = 2x$, give your answers correct to three decimal places

b) $\cos(x) = \sin(2x)$, give your answers as exact answers.

Question 8

If $x \in \left[\frac{3\pi}{2}, 2\pi\right]$ and $\csc(x) = -\frac{2}{\sqrt{3}}$, then find the exact value of:

a) $\cos(x)$

b) $\cos^2(x) + \tan^2(x)$

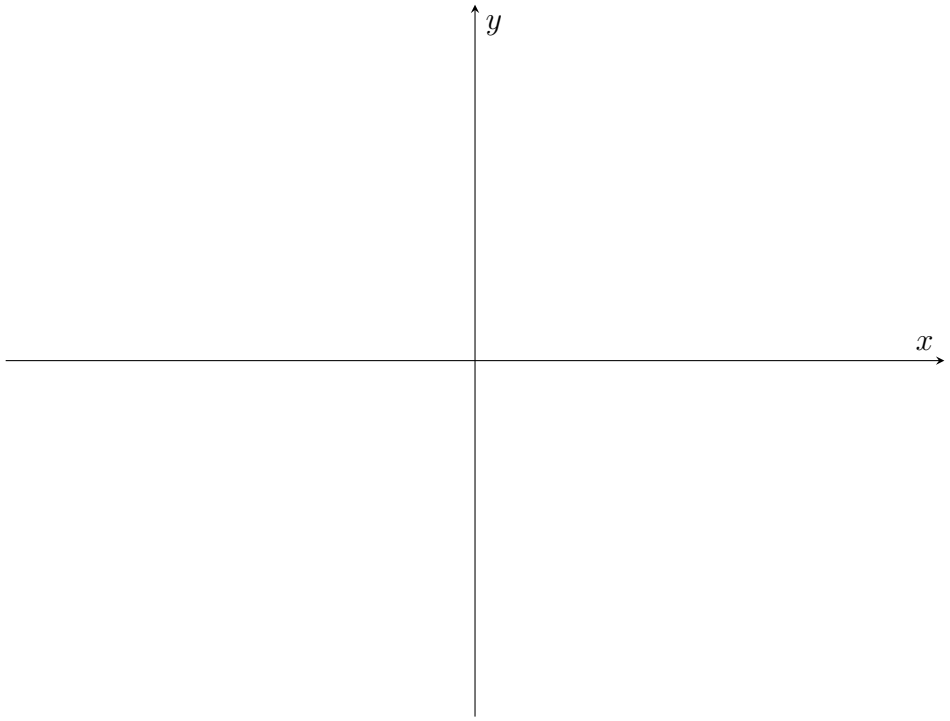
c) $\cot(x)$

Question 9

Consider the function $f(x) = 2\cos^{-1}(x+4) - \dfrac{\pi}{2}$

a) State the implied domain and range of f

b) Sketch the graph of the function f over its implied domain, labelling all key features.



Question 9

Find the domain and range of the following:

a) $y = \sin(3\tan^{-1}(2x))$

b) $y = \cos^{-1}(x^2 - 3x))$

Question 11

Find in simplest surd form:

a) $\sec\left(\dfrac{11\pi}{12}\right)$

b) $\csc(4\theta)$, if $\dfrac{\pi}{2} < \theta < \pi$, and $\tan(\theta) = -2\sqrt{2}$

c) $\tan\left(\dfrac{\theta}{2}\right)$, if $\dfrac{\pi}{2} < \theta < \pi$, and $\csc(\theta) = 1.2$

d) $\sin\left(\tan^{-1}\left(\cos\left(\sin^{-1}\left(-\dfrac{2}{3}\right)\right)\right)\right)$

e) Range of $y = -\dfrac{1}{\sqrt{2}} \sec\left(x - \dfrac{\pi}{6}\right) - \sqrt{3}$

Question 12

Show that $\sin^{-1}\left(\dfrac{12}{13}\right) - \sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right) = \sin^{-1}\left(\dfrac{7\sqrt{2}}{26}\right)$

Question 13

a) Show that $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$

b) Use the result in part **a** to show that $\dfrac{\cos(3\theta)}{2\cos(2\theta) - 1} = \cos(\theta)$

c) Hence, show that $\cos\left(\dfrac{\pi}{12}\right) = \dfrac{\sqrt{6} + \sqrt{2}}{4}$.

Question 14

Find the exact value of $\cos\left(\sin^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{3}{2}\right)\right)$

Question 15

a) Find the exact value of $\cot\left(-\frac{\pi}{12}\right)$ in the form $a+\sqrt{b}, a, b \in \mathbb{R}$.

b) **i.** Show that the equation $\cos\left(\frac{\cos(2\theta)}{\cos(\theta)+\sin(\theta)}\right)=\frac{1}{2}$ can be written as $\sin(2\theta)=\cos(2\theta)$

ii. Solve $\sin(2\theta)=\cos(2\theta)$ for $-\pi \leq \theta < \pi$

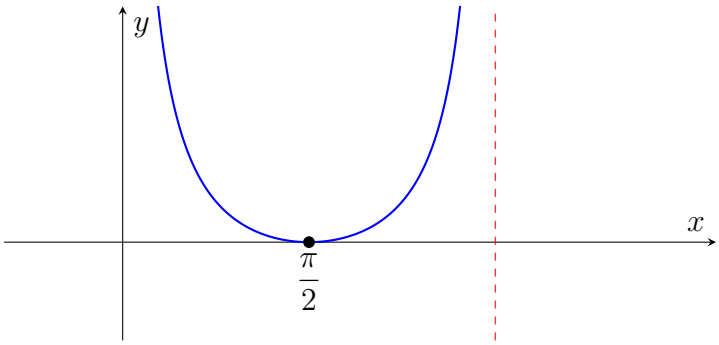
Question 16

a) Find x if $\tan^{-1}(\sin(x)) = -\frac{\pi}{4}, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

b) Show that $\sec\left(\frac{5\pi}{12}\right) = \sqrt{6} + \sqrt{2}$

Question 17

The cross section of a segment of a rollercoaster is shown below.



If the cross-section is modelled by the curve with equation $y = a\csc(x) - b$ and the curve passes through the points $\left(\frac{\pi}{6}, 2\right)$ and $\left(\frac{\pi}{2}, 0\right)$, then find the values of a and b.

Question 18

For the equation $\csc(2x) = \sec(4x), x \in \left(0, \frac{\pi}{2}\right)$

a) **i.** Write expression $\csc(2x)$ as a function of sine only.

ii. Write the expression $\sec(4x)$ as a function of sine only.

b) Hence, solve $\csc(2x) = \sec(4x)$ for x.

Question 19

Find the domain and range of:

a) $y = \sin(3\tan^{-1}(2x))$

b) $y = \cos^{-1}(x^2 - 3x)$

Question 20

a) If $\sec(A) = -3$ and $A \in [\frac{\pi}{2}, \pi]$

i. Show that $\sin(A) = \frac{2\sqrt{2}}{3}$

ii. Therefore find $\cot(A)$

b) If $y = \sec\left(3x - \frac{\pi}{2}\right)$ where $x \in [-\pi, \pi]$ identify the asymptotes.

c) Prove $\sec(2x) = \frac{\cos(x) + \sin(x)}{\cos(x) - \sin(x)} - \tan(2x)$.

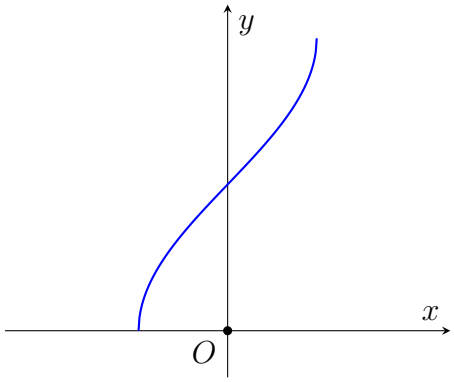
Question 21

a) Show that $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$

b) Hence, find the general solution to the equation $\cos(3\theta) + cos(\theta) = 0$

Question 22

The graph shows the function with rule:
$y = a\sin^{-1}(bx) + c, \qquad$ where $a, b, c \in \mathbb{R}$.



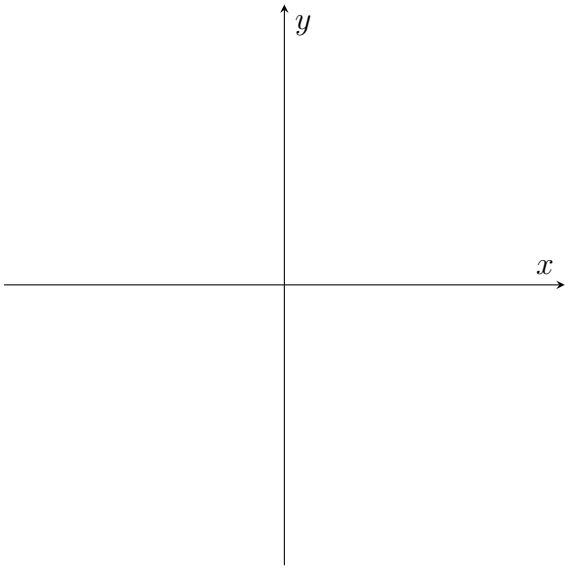
a) **i.** Given that the graph is symmetrical about the point $\left(0, \frac{3\pi}{2}\right)$, find the values of a and c.

ii. Given that in addition, point $\left(\frac{1}{4}, 2\pi\right)$ lies on the graph, show that $b = 2$.

iii. Hence state the domain and range of the function.

b) **i.** Find the rule of the inverse function.

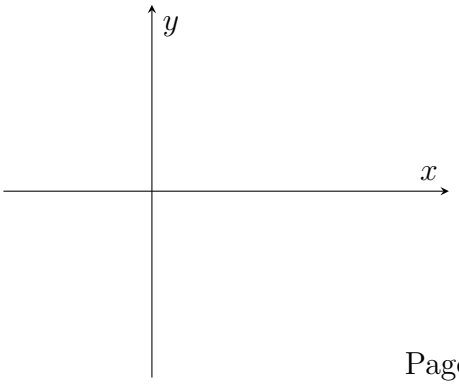
ii. Express the rule for the inverse in the form $f(x) = m\cos(nx)$, and hence sketch the graph of the inverse on the axis below.



Question 23

a) Solve $\csc\left(x - \frac{\pi}{3}\right) + 2 = 0, x \in [0, \pi]$

b) Graph $y = \csc\left(x - \frac{\pi}{3}\right) + 2, x \in [0, \pi]$.
Labelling all key features



Question 24

a) Show that $\tan\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

b) Given that $\sin(2\theta - \alpha) = \lambda\cos(2\theta + \alpha)$, show that $\tan(2\theta) = \frac{\lambda + \tan\alpha}{1 + \lambda\tan\alpha}$

c) Hence or otherwise, find the solution(s) for $0 < \theta < \pi$ of the equation $\sin(2\theta - \alpha) = \lambda\cos(2\theta + \alpha)$, when $\alpha = \frac{\pi}{6}$ and $\lambda = \frac{1}{2}(1 - \sqrt{3})$

Multiple Choice Questions

Question 1

If $\sin\theta = \dfrac{3}{4}$ and given $\dfrac{\pi}{2} < \theta < \pi$, then $\sin\left(\theta + \dfrac{\pi}{3}\right)$ equals

A. $\dfrac{3+\sqrt{21}}{8}$

B. $\dfrac{4+3\sqrt{3}}{10}$

C. $\dfrac{9}{8}$

D. $\dfrac{4-3\sqrt{3}}{10}$

E. $\dfrac{3-\sqrt{21}}{8}$

Question 2

The number of distinct solutions of the equation $x\sin(x)\sec(2x) = 0, x \in [0, 2\pi]$ is:

A. 3

B. 4

C. 2

D. 5

E. 6

Question 3

Consider the function f with rule $f(x) = a\cos^{-1}(x-b)$. Given that f has domain $[2,4]$ and range $[0, 6\pi]$, it follows that:

A. $a = 6$ and $b = -3$

B. $a = 3$ and $b = 6$

C. $a = -3$ and $b = 6$

D. $a = 6$ and $b = 3$

E. $a = -6$ and $b = 3$

Question 4

The graph of the function $f : [0, \infty) \to \mathbb{R}, f(x) = \csc(ax), a > 0$ has asymptotes located at:

A. $x = 0, \frac{2\pi}{a}, \frac{4\pi}{a}, \frac{6\pi}{a}, ...$

B. $x = 0, \frac{\pi}{2a}, \frac{3\pi}{2a}, \frac{5\pi}{2a}, ...$

C. $x = 0, \frac{\pi}{a}, \frac{2\pi}{a}, \frac{3\pi}{a}, ...$

D. $x = 0, \frac{\pi}{a}, \frac{3\pi}{a}, \frac{5\pi}{a}, ...$

E. $x = 0, \pi a, 2\pi a, 3\pi a, ...$

Question 5

The domain and range of the function $y = 2\sin^{-1}(2x + 1) - \pi$ are given respectively by:

A. $[-\frac{1}{2}, \frac{1}{2}]$ and $[-\pi, \pi]$

B. $[-\pi, \pi]$ and $[-1, 0]$

C. $[0, 1]$ and $[-\pi, 0]$

D. $[-\pi, 0]$ and $[-\frac{1}{2}, 0]$

E. $[-1, 0]$ and $[-2\pi, 0]$

Question 6

The largest domain of $y = \cos^{-1}(\sin(-2x))$ is:

A. $[-\frac{\pi}{2}, 0]$

B. $[0, 2\pi]$

C. $[-\pi, \pi]$

D. $[-\frac{\pi}{4}, \frac{\pi}{4}]$

E. $[-2\pi, 2\pi]$

Question 7

If $\cos A = \frac{2}{5}$ and $\frac{3\pi}{2} < A < 2\pi$, then the exact value of $\sin\left(\frac{A}{2}\right)$ is:

A. $\frac{1}{\sqrt{30}}$

B. $\frac{2\sqrt{5}}{5}$

C. $\frac{4}{5}$

D. $-\frac{2\sqrt{5}}{5}$

E. $\frac{\sqrt{30}}{10}$

Question 8

The graph of $y = -\sec(a(x-b))$ is shown below for $0 \leq x \leq \pi$. The values for a and b could be:

A. $a = 1, b = \frac{\pi}{2}$

B. $a = 1, b = \frac{\pi}{4}$

C. $a = 2, b = \frac{\pi}{2}$

D. $a = 2, b = \frac{\pi}{4}$

E. $a = 2, b = -\frac{\pi}{4}$

Extended Response Questions

Question 1

Consider the function $f(x) = \sin\left(2\tan^{-1}\left(\frac{x}{3}\right)\right)$ where the domain of $\sin(x)$ is restricted to $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

a) Find the exact value of the following. (if no value exists, clearly explain why)

i. $f(3)$

ii. $f(-\sqrt{3})$

iii. $f(3\sqrt{3})$

b) Find the implied domain and range of $f(x)$

c) Find a rule for $f(x) = \dfrac{2ax}{bx^2 + a^2}$ where a and b are real numbers.

d) **i.** Find the exact value of $f(2)$.

ii. Use algebra to find the exact value of $f^{-1}\left(-\frac{3}{5}\right)$ where $f^{-1}(x)$ denotes in inverse function of $f(x)$

e) By considering an appropriate reciprocal function, find the equation of all asymptotes and the coordinates of all axes intercepts and stationary points of the function above in **part d.** above.
Hence sketch the graph of $y = f(x)$