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Specialist Maths Units 3/4

Dynamics

Practice Questions

Short Answer Questions

Question 1

A body of mass 0.5kg moves so that its position vector r at time t seconds is given by $\underset{\sim}{r} = 24t\underset{\sim}{i} + (18t - 5t^2)\underset{\sim}{j}$, where distance is measured in metres. Find the resultant force on the body.

Question 2

A particle of mass 1kg falls from rest through a liquid for which the resistance to motion $\frac{V}{5}$ newtons, where V m/s is the velocity of the particle at time t seconds.

a) Show that the acceleration, $\underset{\sim}{a}$, of the body at time t, in terms of V, is given by $\frac{5g - V}{5}$, where g, where gm/s^2 is the acceleration due to gravity.

b) Find the velocity of the particle at time t seconds.

c) Find the time it takes for the particle to reach a velocity of $\frac{5g}{2}$m/s.

Question 3

A particle rope will break when its tension exceeds $200g$ newtons.
Find in terms of g the exact greatest acceleration with which an 80 kg mass can be pulled vertically upwards by this rope.

Question 4

Two fisherman hook into a large fish of mass M kg. Their fishing lines are taut and at an angle of $60°$. At a specific moment the system is in equilibrium, and one of the fishermen observes that $T = 2\sqrt{3}$ newtons. Calculate the value of M.

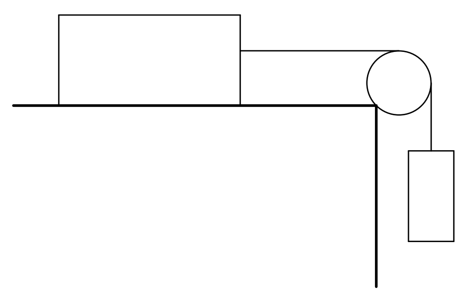
Question 5

A train of mass 50 tonnes is subjected to a resistance force of $2000v$, where v is the velocity in ms^{-1} of the train at time, t seconds. If the velocity of the train is given by $30\left(1 - e^{-\frac{t}{20}}\right)$ ms^{-1}, find the driving force of the engine.

Question 6

The diagram below shows an object of mass 6 kg at rest on a rough horizontal table and a block of mass 2 kg hanging vertically.
The coefficient of friction between the object and the table is 0.4. The object and the block are connected by a light inextensible string which passes over a smooth pulley of negligible mass at the edge of the table.

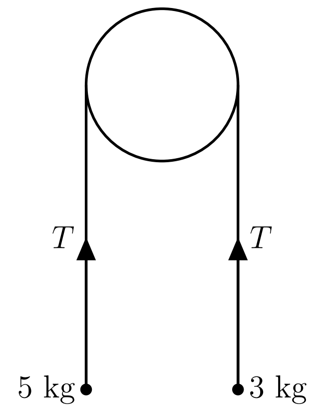
a) Find, in terms of g, the exact size of the friction force acting on the 6 kg mass.



b) Determine the mass of the heaviest object that can be attached to the 2 kg object while the 6 kg mass remains at rest.

Question 7

The diagram shows two particles of mass 3 kg and 5 kg connected by a light string passing over a smooth pulley. The tension in the string is T newtons. Let a ms^{-2} be the magnitude of the acceleration of each particle. Find a and T.



Question 8

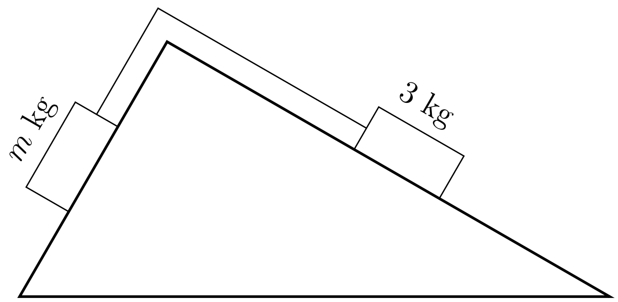
A body of mass 50 kg, initially at rest, is acted upon by a force of 100 newtons for ten seconds. It maintains its velocity for twenty seconds and then slows to a stop uniformly over six seconds. How far has it travelled?

Question 9

A box of mass 8 kg sits on top of another box of mass 10 kg. The 10 kg box lies on the horizontal floor of a lift. The lift is accelerating downwards at 2ms^{-2}. Find in newtons and in terms of g the exact size of the force exerted by the 10 kg box on the lift floor.

Question 10

A light inextensible string passes over a smooth pulley connecting a m kg mass on a smooth frictionless plane inclined plane at 60° to the horizontal and a 3 kg mass on a smooth frictionless inclined plane at 30° to the horizontal as can be seen in the diagram below.



a) Find the tension in the string and the mass of the object if the system is accelerating to the left at $0.5ms^{-2}$ giving your answer in terms of g.

b) The surfaces are replaced with rough panels where the coefficient of friction is 0.1 on both planes. Find the acceleration of the system in ms^{-2} correct to 1 decimal place if the mass, m, is replaced by a mass of 6 kg and the system moves to the left.

Question 11

A particle, P, of mass 6 kg, hangs at the lower end of a taut string, which is attached at its upper end to a fixed point O. The particle is held at rest by a force of magnitude 30 newtons, which acts horizontally on the particle. The string makes an angle θ with the vertical.

a) Draw a force diagram labelling all forces acting on the particle.

b) Find the exact value of the angle θ.

Question 12

A car if mass 1.6 tonnes is moving along a straight horizontal road at a speed of 90km/h when the driver breaks. The braking force is $\frac{369}{100}v^3$ newtons, where v is the speed in m/s after the driver applies the brakes. After travelling t seconds the speed of the car has been reduced to 57.6 km/h and has travelled x metres. Find the exact values of t and x.

Question 13

A flowerpot of mass 6 kg is held in equilibrium by two light ropes, both of which are connected to a ceiling. The connection points are 5m apart. The first rope has a length of 4m and the second rope has a length of 3m.
Find the newtons and in terms of g the exact size of the tension in each rope.

Question 14

A 5 kg mass is placed on a smooth inclined ramp angled at 30° to the horizontal. To stop it from sliding down the ramp, a string secured to a horizontal beam above is attached to the mass. While keeping the mass in place, the taut string makes an angle of 60° to the beam.

a) Find an expression for T, the tension in the string in terms of g.

b) Find the normal reaction force, N, in terms of g.

Question 15

The acceleration, a ms^{-2}, of a particle moving in a straight line is given by a $a = \dfrac{v}{\log_e(v)}$, where $v > 1$, is the velocity of the particle in ms^{-1} at time t seconds. The initial velocity of the particle was 5 ms^{-1}. Find an expression for the velocity of the particle, in terms of t.

Question 16

A 10 kg parcel is placed on a rough surface of a conveyor belt which is inclined at an angle of 30° to the ground. The coefficient of friction between the parcel and the belt is $\dfrac{1}{\sqrt{3}}$. The parcel is pulled up on a conveyor belt with a light inextensible rope with a tension force of 100 newtons.

a) Draw a diagram showing all forces acting on the parcel and label them.

b) Find the acceleration of the parcel up the belt, giving your answer in terms of g.

Question 17

A car of mass 1500 kg moves from rest by the engine exerting a time-dependent force of $96000te^{-\frac{1}{3}}$ newtons, where t is the time in seconds from when the car begins to move. The resistance to the motion is $4500v$ newtons where v is the speed of the car is m/s.

The equation of motion of the car is given by $\dfrac{dv}{dt} = ate^{-\frac{t}{3}} + bv$, where a and b are constant. Find the values of a and b.

Question 18

A large brick of mass 5 kg is accidentally dropped from a high-rise construction site. As it falls vertically downwards it is retarded by a force of $0.01v^2$ newtons where v m/s is the speed of the brick t seconds after it has dropped.

a) Show that x metres, the distance travelled by the brick as it falls is given by the formula.

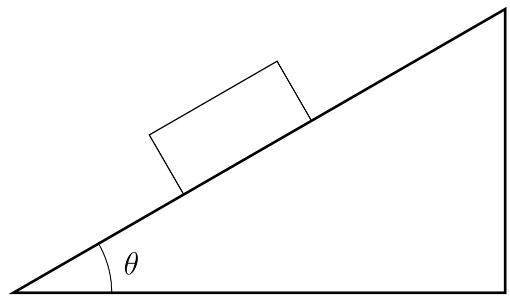
$$x = 250\log_e\left(\frac{4900}{4900 - v^2}\right)$$

b) Find a formula for t in terms of v.

Question 19

A car of mass 1000 kg moves up a roadway inclined at an angle of 30° to the horizontal. The magnitude of frictional forces F acting on the car is one quarter of its weight force, W. The engine exerts a tractive force, P, on the car causing it to move up the slope with an acceleration of $\frac{g}{10}$ ms^{-2}.

a) Label all forces acting on the car on the diagram below.



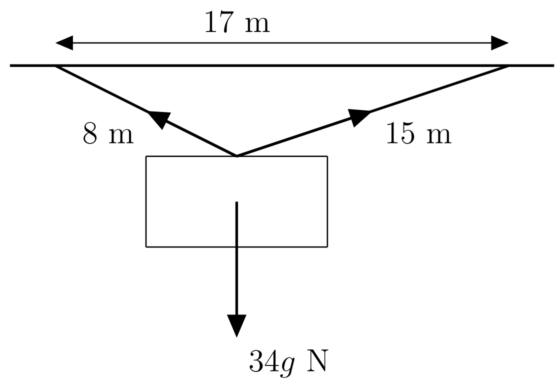
b) Find the magnitude of the tractive force, P, exerted by the engine in terms of g.

Question 20

A particle of mass 6 kg is constrained to move in the line defined by the vector $\underset{\sim}{w} = \underset{\sim}{i} + 2\underset{\sim}{j}$. It is acted upon by two forces, $\underset{\sim}{F_1} = 3\underset{\sim}{i} + \underset{\sim}{j}$ newtons and $\underset{\sim}{F_2} = 2\underset{\sim}{w}$ newtons. The particle accelerates at a m/s^2 in the direction of w. Calculate the value of a.

Question 21

A painting in an art gallery is held in equilibrium by two ropes of different length hung from two points in a on the wall, as shown in the adjacent diagram. The triangle formed has three sides of 8, 15 and 17 metres respectively. Find the tension, in terms of g, in each rope.



Question 22

A cyclist rides on a road with velocity given by $v = \dfrac{20}{\pi}\tan^{-1}(t)$, where $t \geq 0$. The mass of the cyclist including the bike is 100 kg.

a) If a horizontal resistance wind force of 4 newtons is applied to the cyclist 10 seconds after the cyclist commences riding, find the driving force the cyclist applies on the bike in terms of π at this instant.

b) Find the terminal velocity of the cyclist in km/h.

Question 23

Two particles of masses 3 kg and 2 kg are both hanging vertically and are connected by a light inextensible string which passes over a smooth pulley. Initially the 3 kg mass is 1.5 metres above the ground. Find the time take for the 3 kg block to hit the ground once the system is released from rest. Give your answer in terms of g.

Question 24

A car of mass 1400 kg is pulling a caravan of mass 2850 kg up an incline of $\sin^{-1}\left(\frac{1}{4}\right)$. The resistances to the motion of the car and caravan are both one quarter of their weight forces. The car moves up the incline with an acceleration of 1 m/s^2.

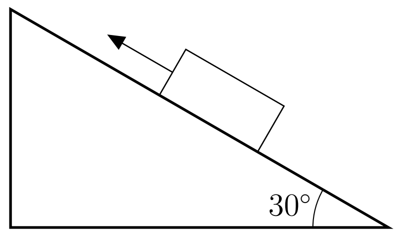
a) Find the tension in the tow bar.

b) Find the tractive force exerted by the engine of the car.

Question 25

Jack drags a 5 kg cart up a 100 m long path that is inclined at an angle of 30° to the horizontal, the coefficient of friction is 0.12 and Jack exerts a force of 10 newtons on the cart.

a) Complete the force diagram below indicating all forces that are applied to the cart as Jack drags it up the inclined plane.



b) Show that the acceleration of the cart up the hill is negative ans state its value correct to 2 decimal places.

c) Using your value for the acceleration found in **part b.**, how far can Jack drag the cart up the hill before it stops? Answer correct to 2 decimal places.

Question 26

A body of mass m kg rests on a rough plane inclined at $a°$ to the horizontal, with coefficient of friction μ. A upwards force of magnitude P is applied at $\theta°$ to the plane. The body is on the point of slipping down the plane.

a) Draw a force diagram showing all forces acting on the body.

b) Find an expression for the least value of P required to prevent the particle from sliding down the plane.

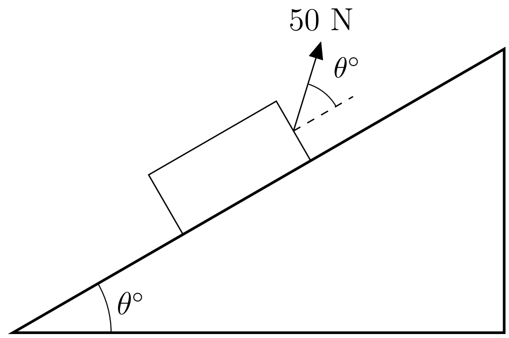
Question 27

A diver fires a spear of mass 150 g from his gun vertically upwards in water at a depth of 10 metres below the surface was an initial velocity of 30 m/s. The retardation from the water is $0.015v^2$ newtons where v is the velocity of the spear t seconds after being shot. Find the velocity of the spear as it breaks the surface. Give your answer in terms of g.

Question 28

A particle of mass 5 kg is being pulled up a plane at $\theta°$ to the horizontal by a force of 50 newtons acting at an angle of $\theta°$ upwards from the plane, as shown. $\theta°$ is an acute angle and $\tan(\theta°) = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{4}$.

a) Find, in terms of g, the magnitude of the normal reaction force of the plane on the particle.



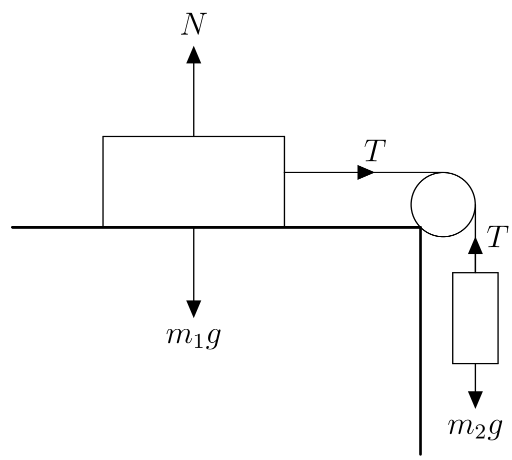
b) Use your answer from **part a.** to calculate the acceleration of the particle up the plane, again giving your answer in terms of g.

Question 29

A particle of mass 2.5 kg moves up a smooth plane inclined at an angle of 30° to the horizontal with an acceleration of a m/s^2 when a force of F newtons acting parallel to the plane is applied. When the force F newtons acts at an angle of 30° to the plane, the move up the plane with an acceleration of $\frac{a}{5}$ m/s^2. Find the value of F.

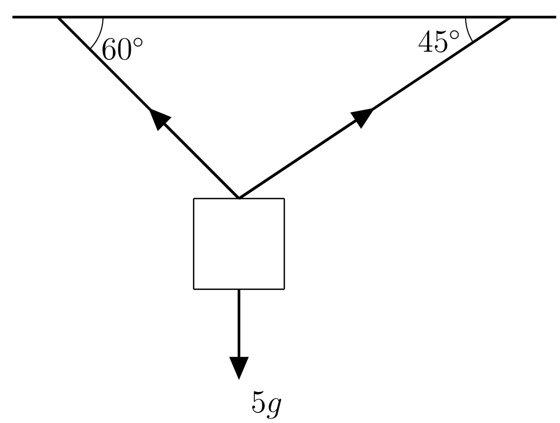
Question 30

Two particle of masses m_1 and m_2 kg are connected by a light inextensible string. The mass m_1 lies on a smooth horizontal table and the other mass m_2 hangs vertically over the edge of the table. The connecting string passes over a smooth pulley at the table's edge. The mass hanging vertically moves downwards and the acceleration of the system is $\frac{7g}{10}$ m/s^2. When 1 kg is added to both masses the acceleration of the system is reduced to $\frac{2g}{3}$ ms^2. Find the values of both masses m_1 and m_2.



Question 32

A mass of 5 kg is supported by two light inextensible strings both attached to a ceiling. One string makes an angle of 60° with the ceiling and the other string makes an angle of 45° with the ceiling. Find the tension in the longer string.



Question 33

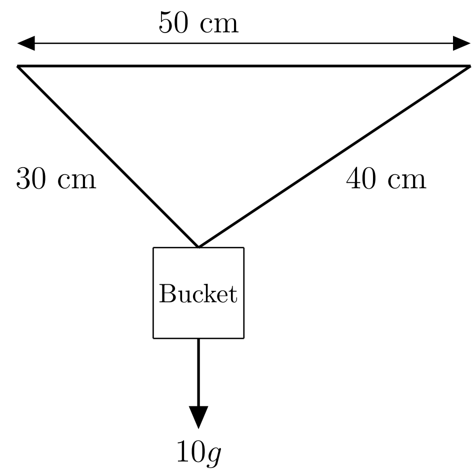
Jack and Jill are retrieving a pail of water from the bottom of a well. A mass M kg sits on a smooth plane inclined at 60° to the horizontal. This mass is connected by a light inextensible string passing over a smooth pulley to the filled pail of water (total mass 10 kg) at the bottom of the well. Jill applies a force of 5 newtons parallel to the plane on the mass M to lift up the pail of water. What mass, M kg is required for the pail to move upwards with an acceleration of 0.2 m/s? Give your answer in terms of g.

Question 34

Jill wants to place the pail of water into the cart, however Jack, in his infinite wisdom decides to push the cart down the hill, without the pail and without them. The hill is inclined at an angle of 30° to the horizontal, the wind and rough surface combined exerts a resistive force of kv where v is the speed in m/s and k is a constant of proportionality.
Find an expression for the velocity, v m/s, of the 5 kg cart in terms of k and t seconds after it has started moving from rest, down the hill.

Question 36

Jack and Jill have to carry the 10 kg pail of water home. They each hold onto the bucket. Jack's arm is 40 cm long and Jill's is 30 cm long and they are walking 50 cm apart.
Find the tension in each arm.



Multiple Choice Questions

Question 1

A body of mass 2 kg is subjected to forces $12\underset{\sim}{i} + 8\underset{\sim}{j}$ N and $-12\underset{\sim}{j}$ N. The magnitude of the body's acceleration, in m/s^2 is:

A. $12\underset{\sim}{i} - 4\underset{\sim}{j}$ N

B. $6\underset{\sim}{i} - 2\underset{\sim}{j}$ N

C. $4\sqrt{10}$

D. $2\sqrt{10}$

E. $6\sqrt{5}$

Question 2

The external resultant force on a body is zero. Which one of the following cannot be true?

A. The body remains stationary.

B. The body moves in circular path.

C. The body moves in a straight line.

D. The body moves with constant velocity.

E. The body's momentum remains constant.

Question 3

A body of mass 15 kg travelling at a constant speed is acted upon by a force of 12 newtons. The speed of the body 4 seconds late is 12 m/s. It follows that the initial momentum of the object was:

A. 324 kgm/s

B. 132 kgm/s

C. 205 kgm/s

D. 36 kgm/s

E. 60 kgm/s

Question 3

Three forces $\underset{\sim}{P}, \underset{\sim}{Q}$ and $\underset{\sim}{R}$ act on a body of mass 8 kg. The forces are all measured in newtons and $\underset{\sim}{P} = 4\underset{\sim}{I} - \underset{\sim}{j}, \underset{\sim}{Q} = -3\underset{\sim}{j} + 2\underset{\sim}{j}$ and $\underset{\sim}{R} = 2\underset{\sim}{I} + 3\underset{\sim}{j}$. The magnitude of the acceleration of the body in m/s^2 is:

A. $\frac{5}{8}$

B. $\frac{3}{4}$

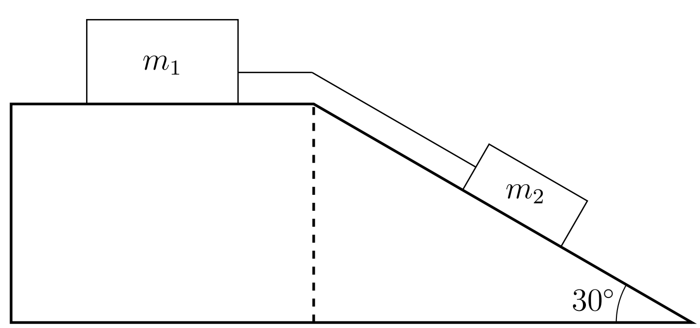
C. $\frac{7}{8}$

D. $\frac{4}{3}$

E. $\frac{5}{2}$

Question 4

A mass of m_1 kg rests on a rough horizontal surface and is connected by a light string that passes over a smooth pulley and is connected to a second mass m_2 kg. This seconds mass rests on a plane with a rough surface inclined at an angle of 30° to the horizontal.
The coefficient of friction between the m_1 kg mass and the horizontal surface is μ as is the coefficient of friction between the m_2 kg mass and the inclined plane. The $2m$ kg mass is on the point of slipping down the plane. The ratio $\frac{m_2}{m_1}$ is equal to:



A. $\frac{1}{2\mu}$

B. $\frac{1-\sqrt{3}\mu}{2\mu}$

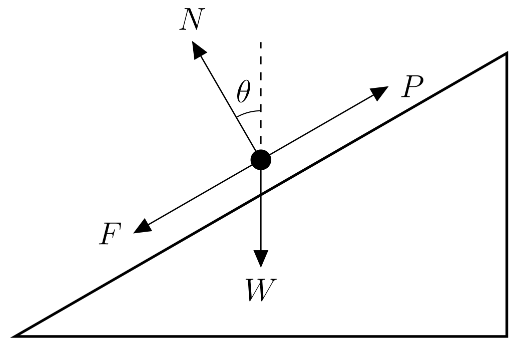
C. $\frac{1+\sqrt{3}\mu}{2\mu}$

D. $\frac{\sqrt{3}-\mu}{2\mu}$

E. $\frac{\sqrt{3}+\mu}{2\mu}$

Question 5

A particle is on the verge of moving up an inclined plane. It is acted on by four forces, W, N, F and P. Which of the following is true?



A. $P = W\sin(\theta) - F$

B. $P = F + W\sin(\theta)$

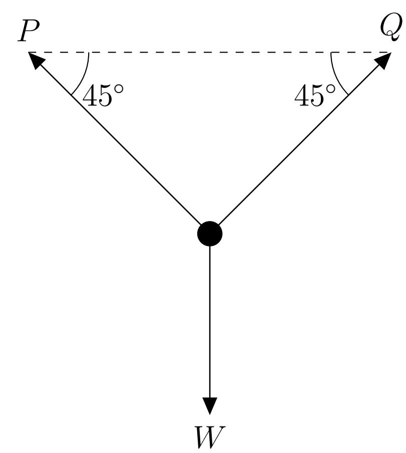
C. $P = F$

D. $N = W\sin(\theta)$

E. $W = N\cos(\theta)$

Question 6

A body is held in equilibrium by 2 light strings attached to the ceiling of a room at two points on the same horizontal level. The 2 strings are at an angle of 45° to the ceiling.
If the forces on the body due to the tension in the strings are given by the vectors $\underset{\sim}{P}$ and $\underset{\sim}{Q}$ and the weight force is $\underset{\sim}{W}$, the which of the following is true?



A. $\underset{\sim}{Q} + \underset{\sim}{P} = \underset{\sim}{W}$

B. $\underset{\sim}{Q} + \underset{\sim}{P} = \sqrt{2}\underset{\sim}{W}$

C. $\sqrt{2}\underset{\sim}{Q} + \sqrt{2}\underset{\sim}{P} = \underset{\sim}{W}$

D. $\underset{\sim}{Q} + \underset{\sim}{P} = -\underset{\sim}{W}$

E. $\underset{\sim}{Q} = \underset{\sim}{P}$

Question 7

A person of mass 80 kg is standing in a lift which is moving with a downward acceleration of magnitude 2 m/s^2. The magnitude of the force in newtons exerted by the floor of the lift on the person is:

A. 160

B. $80g$

C. $180g$

D. $80(g + 2)$

E. $80(g - 2)$

Question 8

A mass of m kg is connected to a mass of M kg, where $M > m$, by a light inelastic string which passes over a smooth pulley. If the acceleration is g m/s^2 and the magnitude of the acceleration of each mass is a m/s^2, then a equals:

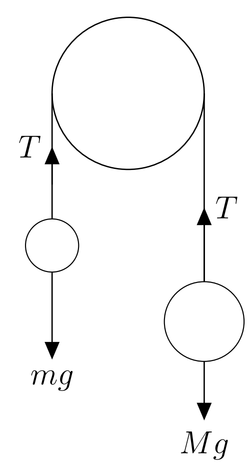
A. $(M - m)g$

B. $\dfrac{(M - m)}{(M + m)}g$

C. $\dfrac{(m - M)}{(M + m)}g$

D. $(M + m)g$

E. $\dfrac{(M + m)}{(M - m)}g$



Question 9

A particle of mass 0.5 kg is subjected to a force of $4\underset{\sim}{i} - 3\underset{\sim}{j}$ newtons. The magnitude of the acceleration in m/s^2 is:

A. 1.5

B. 2

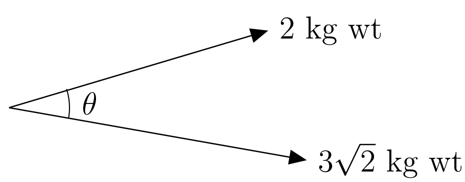
C. 6

D. 2.5

E. 10

Question 10

Two forces of magnitude $3\sqrt{2}$ kg wt and 2 kg wt have a resultant force of $\sqrt{34}$ kg wt.



The magnitude of the angle, θ, between them is:

A. 45°

B. 30°

C. 60°

D. 90°

E. 135°

Question 11

A cupboard of mass 100 kg stands on a rough floor with a coefficient of friction of 0.25. Jo exerts a horizontal force of 200N on the cupboard. If F N is the magnitude of the frictional force then the cupboard.

A. Does not move because $F < 200$.

B. Moves because $F < 200$.

C. Does not move because $F = 200$.

D. Moves because $F > 200$.

E. Does not move because $F > 200$.

Question 12

A business person holding a briefcase is standing in a lift accelerating upwards at 1 m/s^2. If the tension in the person's arm is 27 N, the mass of the briefcase, in kg is:

A. 1.5

B. 2.0

C. 2.5

D. 3.0

E. 3.5

Question 13

A truck of mass 1.6 tonnes is moving at constant speed and covers 450 metres in one minute. Its momentum in kgm/s is:

A. 12

B. 720

C. 12000

D. 43200

E. 720000

Question 14

A particle of mass 3 kg is moving with velocity $2\underset{\sim}{i}+3\underset{\sim}{j}$ m/s. Two seconds later, the velocity of the particle is $5\underset{\sim}{i}-\underset{\sim}{j}$ m/s. If the force that caused this change in velocity is assumed to be constant, then (in newtons) it is equal to:

A. $\frac{1}{2}(3\underset{\sim}{i}-4\underset{\sim}{j})$

B. $\frac{3}{2}(3\underset{\sim}{i}-4\underset{\sim}{j})$

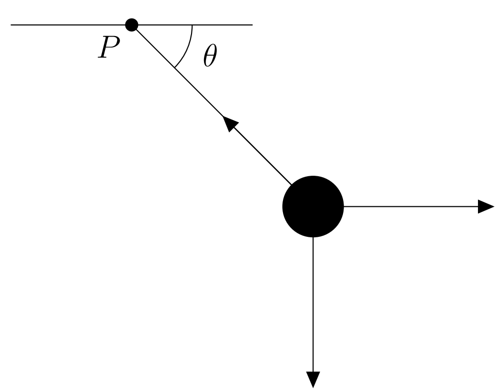
C. $\frac{1}{2}(3\underset{\sim}{i}+2\underset{\sim}{j})$

D. $\frac{1}{3}(3\underset{\sim}{i}+2\underset{\sim}{j})$

E. $\frac{3}{2}(3\underset{\sim}{i}+4\underset{\sim}{j})$

Question 15

A swing seat S is attached to the end of a light cable which is fixed at the other end to a point P on the ceiling. The force on S, due to the tension in the cable is $\underset{\sim}{T}$ and its weight force is $\underset{\sim}{W}$. When a horizontal force $\underset{\sim}{F}$ is applied to S it is held in equilibrium with the cable inclined at 45° to the ceiling as shown in the diagram. Which of the following is true?



A. $\underset{\sim}{T}=\sqrt{2}\underset{\sim}{W}$

B. $\underset{\sim}{F}+\underset{\sim}{W}-\underset{\sim}{T}=0$

C. $\underset{\sim}{F}+\underset{\sim}{W}+\underset{\sim}{T}=0$

D. $\underset{\sim}{T}=\sqrt{2}\underset{\sim}{F}$

E. $\underset{\sim}{F}=\underset{\sim}{W}$

Question 16

Samantha pulls a toy of mass 2 kg along a level floor using a horizontal, light string attached to the toy. The toy slides over the floor with a constant speed and Samantha's pull has a magnitude of 10 newtons. The coefficient of friction between the toy and the floor is closest to:

A. 0

B. 0.2

C. 0.5

D. 2

E. 5

Question 17

A body of mass 2 kg moves so that its velocity vector $\underset{\sim}{r}$, at time t seconds, is given by $\underset{\sim}{r} - 2\sin(t)\underset{\sim}{i} + 2\sqrt{3}\cos(2t)\underset{\sim}{j}$. At time $t = \frac{\pi}{3}$, the magnitude of the resultant force, in newtons, acting on the body is equal to:

A. $\sqrt{37}$

B. $2\sqrt{37}$

C. $2\sqrt{35}$

D. $\sqrt{35}$

E. 14

Question 18

A box of mass 5 kg rests on a rough horizontal floor with coefficient of friction of 0.3. The box is acted upon by a force P N, inclined at an angle of $30°$ to the horizontal. The smallest value of P that will cause the box to move, correct to 2 decimal places is:

A. 14.47

B. 16.97

C. 20.53

D. 61.20

E. 32.62

Question 21

A force, F acting on an object of mass 2 kg is equal to $F(v) = 2v^2 - 1$ newtons. The object has a velocity of 1 ms^{-1} when it is 1 metre to the right of the origin. The velocity of the object, in terms of the displacement, x, is defined as:

A. $\pm\sqrt{\frac{1}{2}\left(e^{2(x-1)} + 1\right)}$

B. $\frac{1}{2}\log_e|2x^2 - 1| + 1$

C. $\sqrt{\frac{1}{2}\left(e^{2(x-1)} + 1\right)}$

D. $-\left(\frac{1}{2}\log_e|2x^2 - 1| + 1\right)$

E. $-\sqrt{\frac{1}{2}\left(e^{2(x-1)} + 1\right)}$

Question 22

A m kg parcel is on the floor of a lift that is accelerating downwards at $\frac{g}{5}$ m/s^2. The reaction, in newtons, of the floor of the lift on the parcel is:

A. $\frac{g}{5}$

B. $\frac{mg}{5}$

C. $\frac{2mg}{5}$

D. $\frac{3mg}{5}$

E. $\frac{4mg}{5}$

Question 23

Forces $\underset{\sim}{F_1}, \underset{\sim}{F_2}$ and $\underset{\sim}{F_3}$ act on a body. If $\underset{\sim}{F_1} = \underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}, \underset{\sim}{F_2} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and the acceleration of the body is $\underset{\sim}{i} + \underset{\sim}{j}$, which one of the following could $\underset{\sim}{F_3}$ equal ?

A. $-3\underset{\sim}{i} - 2\underset{\sim}{j}$

B. $-3\underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k}$

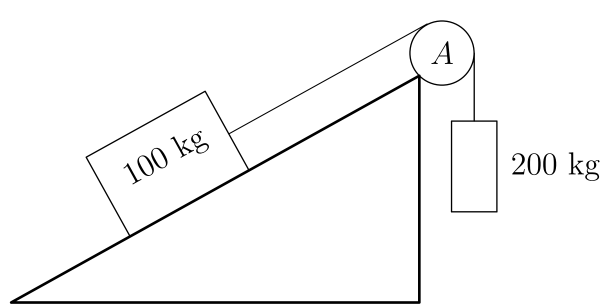
C. $2\underset{\sim}{i} - 3\underset{\sim}{j} - 3\underset{\sim}{k}$

D. $2\underset{\sim}{i} + 3\underset{\sim}{j} + 3\underset{\sim}{k}$

E. $2\underset{\sim}{i} + 4\underset{\sim}{j} + 3\underset{\sim}{k}$

Question 24

The diagram below shows a crate of mass 100 kg on a rough inclined slope and a block of mass 200 kg hanging vertically 50 metres above the ground. The coefficient of friction between the slope and the 100 kg crate is μ. The crate and the block are joined by a light inelastic rope which passes through a smooth pulley at A. The two blocks are in static equilibrium.
The magnitude of the friction force between the crate and the slope is:



A. $100g(2 - \sin(\theta))$

B. $100g(2 - \cos(\theta))$

C. $100\mu g \sin(\theta)$

D. $100\mu g \cos(\theta)$

E. $200g$

Question 25

Consider three forces $\underset{\sim}{F_1} = \sqrt{3}\underset{\sim}{i} + \frac{\sqrt{5}}{2}\underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{F_2} = -2\sqrt{3}\underset{\sim}{i} + 2\sqrt{5}\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{F_3} = \frac{\sqrt{3}}{2}\underset{\sim}{i} - \frac{3\sqrt{5}}{2}utj + 3\underset{\sim}{k}$. The magnitude of the sum of the three forces is equal to:

A. $\frac{3}{2}$

B. 3

C. $\frac{2\sqrt{3}}{2}$

D. $\frac{3\sqrt{3}}{2}$

E. $\sqrt{6}$

Question 26

A body of mass 4 kg slides from rest down an inclined plane of length 3 m. It takes 2 seconds to slide down the plane, the body's momentum at the bottom of the plane, in kgm/s is:

A. 8

B. 12

C. 24

D. 36

E. 48

Question 27

A particle of mass 3 kg travels in a straight line with velocity v m/s and its displacement is x metres. If $v = 3\log_e(3x)$ for $x > 0$, then the maximum force in newtons acting on the particle is closest to:

A. 3

B. 10

C. 24

D. 30

E. 90

Question 28

The diagram shows a particle of mass m_1 kg connected by a light inelastic string which passes over a smooth pulley to a particle of mass m_2 kg, where $m_1 > m_2$. The magnitude and direction of the acceleration of the particle with mass m_1 is:

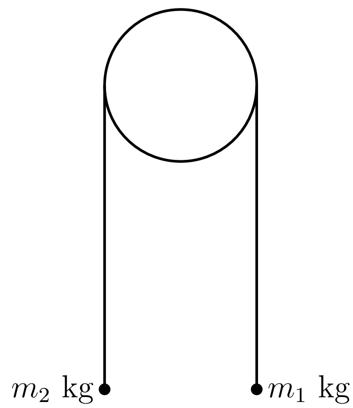
A. $\dfrac{g(m_1 - m_2)}{m_1 + m_2}$ downwards

B. $\dfrac{g(m_1 - m_2)}{m_1 + m_2}$ upwards

C. $\dfrac{g(m_1 + m_2)}{m_1 - m_2}$ downwards

D. $\dfrac{g(m_1 + m_2)}{m_1 - m_2}$ upwards

E. $\dfrac{g(m_1 - m_2)}{m_1 m_2}$ upwards



Question 29

A parachutist of mass m kg falls towards Earth and is slowed by air resistance kx^2 newtons, where x is the displacement of the parachutist t seconds after commencing the fall. An equation of motion for the parachutist is:

A. $\dfrac{dv}{dt} = g - kx^2$

B. $m\dfrac{d^2x}{dt^2} = g - 2kx^2$

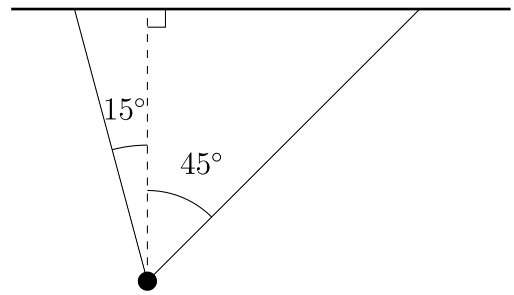
C. $\dfrac{dv}{dx} = \dfrac{g}{v} - \dfrac{2kx^2}{m}$

D. $\dfrac{dv}{dx} = \dfrac{g}{v} + \dfrac{2kx^2}{m}$

E. $\dfrac{d(v^2)}{dx} = 2g - \dfrac{2kx^2}{m}$

Question 30

A mass of 12 kg is suspended from a horizontal ceiling by two light inextensible strings of different lengths. The stronger string makes an angle of 15° with the vertical and the longer string makes an angle of 45° with the vertical as shown in the diagram below. The magnitude in newtons of the tension in the shorter string is:



A. $\sqrt{6}$

B. $4\sqrt{6}$

C. $\sqrt{6}g$

D. $4\sqrt{6}g$

E. $6\sqrt{6}g$

Question 31

A particle of mass 2 kg is being dragged across a rough horizontal surface by a force of magnitude 20 newtons acting at an angle of 30° to the horizontal. The acceleration of the particle is 3 m/s^2. The coefficient of friction between the particle and the rough surface is equal to:

A. $\dfrac{10}{g}$

B. $\dfrac{5}{g-7}$

C. $\dfrac{7}{g-5\sqrt{3}}$

D. $\dfrac{5\sqrt{3}-3}{g}$

E. $\dfrac{5\sqrt{3}-4}{g-5}$

Question 32

A car of mass 1.4 tonnes pulls a trailer of mass 0.3 tonnes at a constant speed up a road inclined at angle θ where $\sin(\theta) = 0.6$. The resistance force to the motion of the car and the trailer is one-tenth of their weight. What is the tension in the tow bar connecting the car to the trailer?

A. 2058 N

B. 294 N

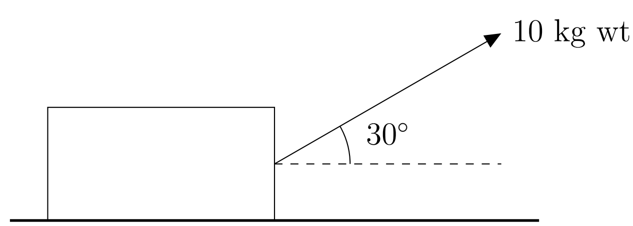
C. 1940.4 N

D. 10290 N

E. 1764 N

Question 33

A box of mass 20 kg is at rest on a horizontal plane. A force of magnitude 10 kg wt acting at an angle of 30° to the horizontal is applied to the block. For equilibrium to be maintained, the coefficient of friction between the box and the plane must be:



A. Less than $\dfrac{\sqrt{3}}{4g}$

B. Less than $\dfrac{\sqrt{3}}{3}$

C. At least $\dfrac{\sqrt{3}}{3}$

D. At least $\dfrac{\sqrt{3}}{4g}$

E. At least $\dfrac{\sqrt{3}}{4}$

Question 34

A ball of mass 0.4 kg is rolled up a smooth hill which is inclined at 30° to the horizontal. If the ball is given an initial velocity of 4 m/s, the distance, in metres, the ball travels up the hill before coming to rest is approximately equal to:

A. 1.4

B. 2.2

C. 2.8

D. 3.0

E. 3.7

Question 35

A body is projected with speed U up a rough plane with coefficient of friction μ and inclination a degrees to the horizontal. The distance the body travels up the plane is equal to:

A. $\dfrac{U^2}{2g(\mu\cos(a)+\sin(a))}$

B. $\dfrac{U^2}{2g(\mu\sin(a)+\cos(a))}$

C. $\dfrac{U^2}{2g(\mu\sin(a)-\cos(a))}$

D. $\dfrac{U^2}{2g(\mu\cos(a)-\sin(a))}$

E. $\dfrac{U^2}{g(\mu\cos(a)+\sin(a))}$

Question 36

A mass 2 kg which is on a rough, horizontal table is connected to a mass of M kg, which is hanging vertically, by means of a light inextensible string which passes over a smooth pulley. If the coefficient of friction between the 2 kg mass and the table is 0.5, and the system has an acceleration of $\frac{g}{3}$ m/s^2, then M is equal to:

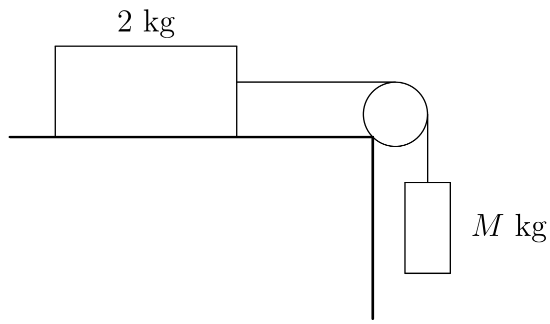
A. 2.3 kg

B. 2.5 kg

C. 2.7 kg

D. 2.9 kg

E. 3.1 kg



Question 37

A stone of mass 10 kg is released from rest from the top of a cliff. While falling, the stone experiences a resistive force that is proportional to its speed. If the constant of proportionality is 2.5, the time taken for the stone to reach a speed of 20ms^{-1}, correct to 2 decimal places is:

A. 2.85 seconds

B. 1.65 seconds

C. 0.91 seconds

D. 1.18 seconds

E. 2.09 seconds

Question 38

A particle of mass m kg, slides down a smooth plane inclined at $\theta°$ to the horizontal. The particle's acceleration, in m/s^2 is:

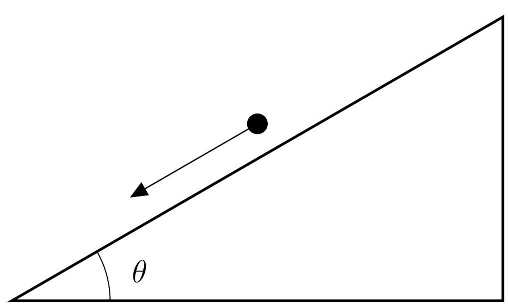
A. g

B. $g\cos(\theta)$

C. $g\sin(\theta)$

D. $mg\cos(\theta)$

E. $mg\sin(\theta)$



Question 39

Particles of mass 2 kg and 5 kg are attached to the ends of a light inextensible string that passes over a fixed smooth pulley as shown. The system is released from rest. Assuming the system remains connected, the speed of the 5 kg mass after two seconds is:

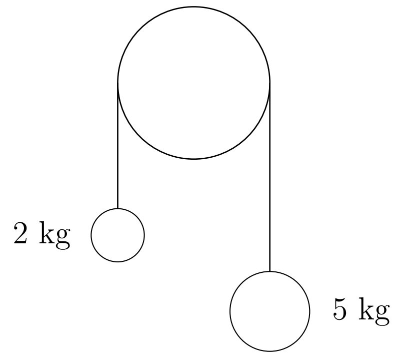
A. 4.0 m/s

B. 4.9 m/s

C. 8.4 m/s

D. 10.0 m/s

E. 19.6 m/s



Question 40

Two particles A and B, move from the same point in a straight line with their respective velocities $V_A = f(t)$ and $V_B = g(t)$ in m/s, where t is in seconds and $t \geq 0$. The velocity time graphs for the particles are shown on the right.
At $t = 50$ seconds, the distance, in metres, between the two particles is:

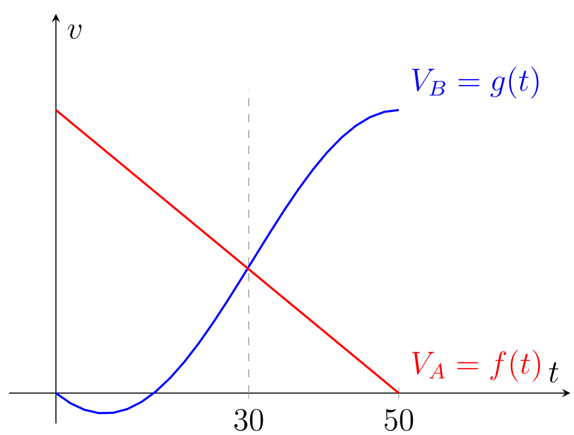
A. $\left| \int_{20}^{50} g(t)\ dt - \int_{0}^{20} g(t)\ dt \right| - 50f(t)$

B. $\left| \int_{0}^{50} g(t)\ dt - 50f(t) \right|$

C. $\left| \int_{0}^{50} (g(t) - f(t))\ dt \right|$

D. $\left| \int_{20}^{50} g(t)\ dt - \int_{0}^{20} g(t)\ dt - 50f(t) \right|$

E. $\left| \int_{30}^{50} g(t)\ dt - \int_{0}^{30} g(t)\ dt - 50f(t) \right|$



Question 41

A body of mass 5.0 kg rests on a plane of inclination 15° in equilibrium limited by friction. If the inclination is increased to 30°, the exact value of the acceleration, in m/s^2, down the plane is:

A. $g - \frac{\sqrt{3}\tan(15°)}{2}$

B. $\frac{g}{2} - \frac{\sqrt{3}\tan(15°)}{2}$

C. $\frac{g}{2} - \frac{\sqrt{3}\sin(15°)}{2}$

D. $\frac{g}{2} - \frac{\sqrt{3}\tan(30°)}{2}$

E. $\frac{\sqrt{3}g}{2} - \frac{\tan(15°)}{2}$

Question 42

The diagram below shoes objects of mass 6 kg and 7 kg attached to end of a light inextensible string that passes over a smooth pulley of negligible mass. The system is released from rest, Assuming the system remains connected, the magnitude of displacement of the 6 kg mass after 2 seconds is:

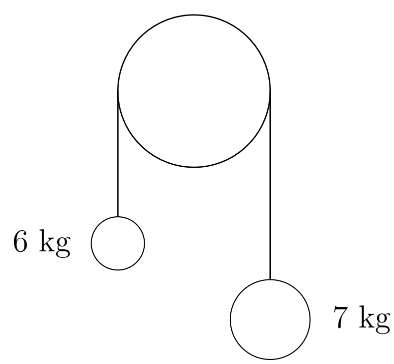
A. $\frac{2g}{131}$

B. $\frac{g}{13}$

C. $\frac{g}{7}$

D. $\frac{g}{6}$

E. $\frac{g}{3}$



Question 43

A block of mass m has an initial velocity of 5 m/s as it slides across the floor. The block comes to rest over a distance of 3 m. The coefficient of friction is:

A. 0.1

B. $\dfrac{25}{6g}$

C. $\dfrac{50}{3mg}$

D. $\dfrac{32}{5g}$

E. $\dfrac{15}{mg}$

Question 44

The velocity vector of a m kg mass moving in the Cartesian plane is given by $\underset{\sim}{v}(t) = 3\cos(2t)\underset{\sim}{i} + 4\sin(2t)\underset{\sim}{j}$ where velocity components are measured is m/s. During its motion, the maximum magnitude of the net force, in newtons, acting on the mass is:

A. 8

B. $8m$

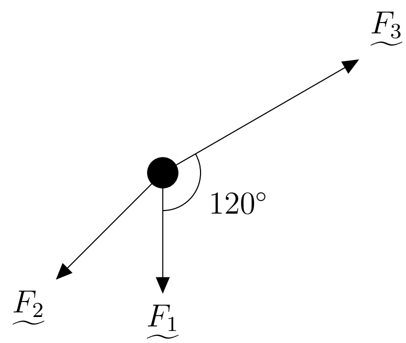
C. $2\sqrt{2}m$

D. $40m$

E. $64m$

Question 45

A body at rest on a horizontal smooth plane is acted upon by four forces $\underset{\sim}{F_1}, \underset{\sim}{F_2}, \underset{\sim}{F_3}, \underset{\sim}{F_4}$. $|\underset{\sim}{F_1}| = 1, |\underset{\sim}{F_2}| = 2$ and $|\underset{\sim}{F_3}| = 3$. The force $\underset{\sim}{F_1}$ acts in a southerly direction and the force $\underset{\sim}{F_2}$ acts in a south-westerly direction.



The magnitude and direction of $\underset{\sim}{F_4}$ is closest to:

A. 1.5 and $N52°W$

B. 0.7 and $N12°E$

C. 2.1 and $N64°W$

D. 0.9 and $N34°W$

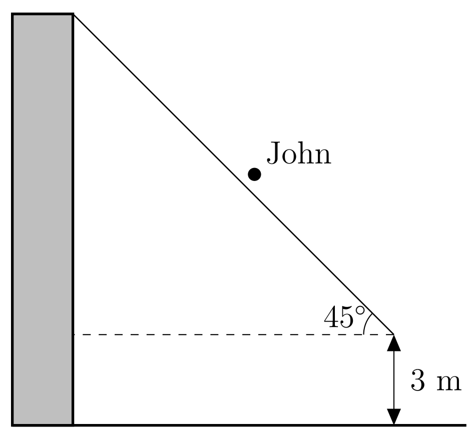
E. 4.0 and $N45°W$

Extended Response Questions

Question 1

At an aquatic fun park, John slides down a straight water slide that is 15 metres long and inclined at an angle of 45° to the horizontal. John has a mass of 40 kg and the coefficient of friction between John and the slide is 0.1 and the end of the slide is 3 metres vertically above the water.

a) Show clearly the forces acting on John on the diagram below



b) Show that the acceleration, a in m/s^2, of John down the slide is given by $\dfrac{9\sqrt{2}g}{20}$.

c) Using the equation of motion of a person sliding down the slide, and assuming that the coefficient of friction is still 0.1, show that $a = \dfrac{9\sqrt{2}g}{20}$ m/s^2 regardless of the person's mass.

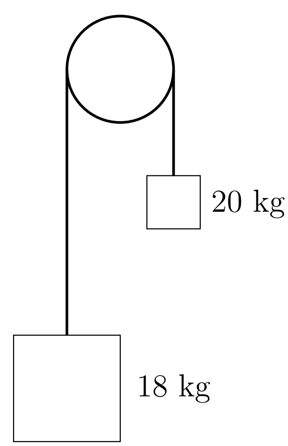
d) Given that John starts from rest at the top of the slide, find correct to 2 decimal places, the total time she is one the slide.

e) Find John's speed at the end of the slide, correct to 2 decimal places.

Question 2

The diagram shows two small blocks of 18 kg and 20 kg connected by a light inextensible string passing over a smooth pulley.
The tension in the string is T newtons. Let a m/s^2 be the acceleration of the 20 kg block downwards.

a) For each block, write an equation that represents the forces acting on that block.



b) Find the exact value of a.

c) Find the exact value of T in newtons.

The system of blocks is reset on a different pulley that exerts a constant frictional force of 2 newtons on each side of the string.

d) Show that the acceleration of the 20 kg block is $a = \dfrac{g-2}{19}$ m/s^2.

e) One second after the block begins to move, they are beside each other and are both 10 m above the floor. At this point, the string breaks. Show that each block hits the floor with a velocity of 14.0 m/s (1 decimal place)

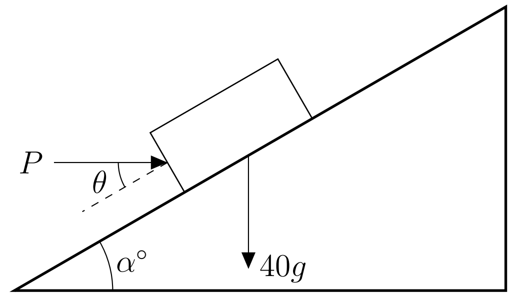
The 20 kg block is moved to a ramp with a rough surface, The coefficient of friction between the block and the ramp is $\mu = \dfrac{1}{4}$. The ramp is inclined at 30° to the horizontal. The 20 kg block accelerates at a m/s^2 down the ramp

f) . Find the value of a.

g) The angle of the ramp to the horizontal is to be changed so that the 20 kg block will remain at rest while on the ramp. Find the largest angle, correct to the nearest 0.1 degrees, for which the block will remain at rest.

Question 3

A body of mass 40 kg is being pushed along at a constant speed up a slope by a rod that is at an angle θ to the plane. The rod exerts a force P on the body.



a) The inclination of the slope to the horizontal is $\alpha°$ where $\sin(\alpha) = \dfrac{3}{5}$ and the coefficient of friction between the body and the surface is 0.3.

i. By resolving the forces parallel and perpendicular to the plane, show that $P = \dfrac{336g\ \text{cosec}(\theta)}{10\cot(\theta) - 3}$.

ii. Sketch the graph of P versus θ for $0 \le \theta \le \tan^{-1}\left(\dfrac{10}{3}\right)$.

iii. What is the significance of the angle $\tan^{-1}\left(\dfrac{10}{3}\right)$ from the graph.

b) A second mass of 10 kg is now attached to the 40 kg mass by a light string and a force of Q newtons parallel to the plane pulls the 40 kg mass up the same inclined plane.

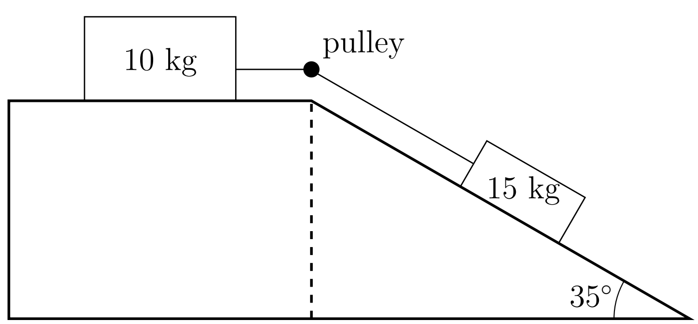
i. Draw a force vector diagram of the new system.

ii. If the pulling force, Q, is 430 newtons, find the acceleration of the two masses up the plane to 3 decimal places.

Question 4

Two bodies of mass 10 kg and 15 kg are connected by a wire passing over a smooth pulley. The horizontal surface is rough ($\mu = 0.2$) and the inclined plane is smooth and inclined at an angle of 35° to the horizontal. The system is released from rest.

a) Show all forces acting on the two bodies.



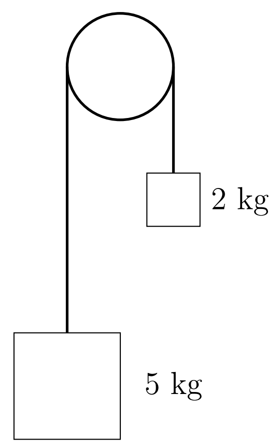
b) Find the acceleration of the system correct to the nearest 0.01 ms^{-2}.

c) The 10 kg body is 5 metres from the pulley. Find the time (to the nearest 0.01 sec) it takes for this body to crash into the pulley.

d) Find the momentum of this body at that time. Give your answer to the nearest kgms^{-1}.

Question 5

The diagram on the right shows particle of mass 5 kg and 2 kg connected by a light inextensible string passing over a smooth pulley of negligible mass. The tension in the string is T newtons.



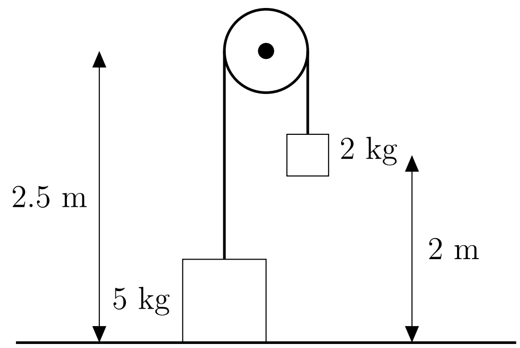
a) Label all forces on the diagram on the right.

b) Find the acceleration of the 5 kg object.

c) Find the value of T.

Question 6

Two particles, A and B, are attached to the ends of a light, inextensible string that passes over a smooth pulley fixed at 2.5 metres above the ground. Initially the string is taut, with A, of mass 0.3 kg, resting on the ground and B, of mass 0.7 kg, supported at a height of 2 metres above the ground. At $t = 0$, B is released and the system moves freely under gravity.



a) Find the tension in the string.

b) Show that B falls $\frac{g}{20}$ metres in the first 0.5 seconds.

c) When $t = 0.5$ seconds part of B becomes detached, leaving B with a mass of 0.2 kg. Find:

i. The velocity of B when it breaks.

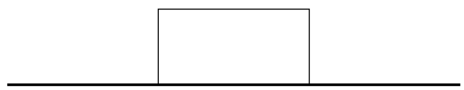
ii. The minimum height above the ground reached by B.

iii. The speed of A as it hits the ground again.

Question 7

Tulio goes on holiday once a year and usually travels to some popular tourist destination. He always travels light with a full suitcase of total mass 20 kg, as he is not healthy, he always drags this behind him.
Tulio drags his suitcase of mass 20 kg over a smooth horizontal floor with a force of magnitude 10 newtons, acting at an angle of 60° to the horizontal.

a) Indicate all forces acting on the suitcase on the diagram below.



b) Find the magnitude of the normal force acting on the surface. Give your answer to the nearest 0.1 newtons.

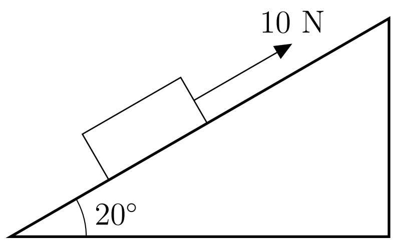
c) Find the acceleration of the suitcase.

d) If the speed of the suitcase was 1.3 ms^{-1} at the start of the smooth floor, how long did it take Tulio to cross the 20 m of smooth floor? Give your answer correct to the nearest 0.1 second.

Question 8

At the end of the 20 m smooth horizontal floor there is a ramp. This ramp is inclined at an angle of 20° to the horizontal. The ramp is rough ($\mu = 0.12$). Tulio drags his suitcase, parallel to the incline plane, with the same force of magnitude 10 newtons.

a) Complete the force diagram on the right, indication all forces that are applied to the suitcase as Tulio drags it up the rough inclined plane.



b) Find the acceleration of the suitcase. Give your answer to the nearest 0.001ms^{-1}.

c) How far can Tulio drag the suitcase up the ramp? Answer to the nearest cm.

Question 9

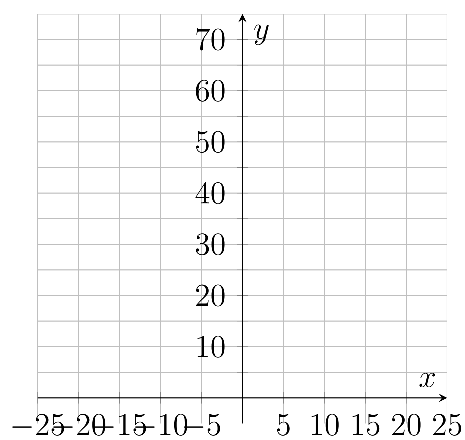
Tulio always goes to a theme park when on vacation. In one such theme park Tulio goes on a ride where his path is described by the following position vector, relative to a point O on the ground.

$$\underset{\sim}{r}(t) = \left(18\sin\left(\frac{\pi t}{20}\right)\right)\underset{\sim}{i} + \left(-18\cos\left(\frac{\pi t}{20}\right) + 20\right)\underset{\sim}{j}, 0 \le t \le 240$$

$\underset{\sim}{i}$ is a unit vector horizontally, $\underset{\sim}{j}$ is a unit vector vertically. Distances is measured in metres and time in seconds.

a) Find the Cartesian equation of the path of the ride.

b) Sketch this path, indicating the starting point and the direction of the ride.



c) Find Tulio's velocity $\underset{\sim}{v}(t)$ and show that Tulio's speed is always constant while on the ride.

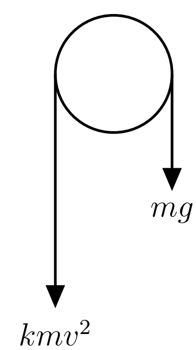
d) If Tulio weights 80 kg wt, find the resultant force $\underset{\sim}{R}(t)$ acting on him throughout the ride.

e) Show that this resultant force is always directed to a fixed point by showing it is always perpendicular to Tulio's velocity.

Question 10

Another attraction at the theme park is a contraption that shoots objects vertically upwards into a dense medium. When Tulio fires an object of mass m kg vertically up into the dense medium. It enters the medium with a velocity of u ms^{-1} and then is subjected to a resistive force of kmv^2 newtons, where v ms^{-1} is the velocity of the object at any position x metres above the entry point and k is a positive constant between 0 and 1.

a) Show that $x = \dfrac{1}{2k} \log_e \left(\dfrac{g + ku^2}{g + kv^2} \right)$.



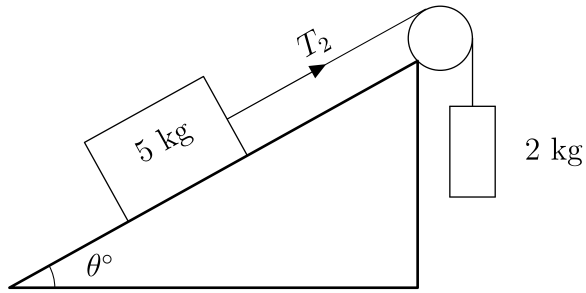
b) When Tulio fires the object into the medium with a velocity of $12ms^{-1}$. It rises to a point of 6.68 m above the entry point before coming to rest. Find the value of k to 3 decimal places.

c) The medium is 10 metres deep. Find the speed that the object must enter the medium to just reach the top. Give your answer to 2 decimal places.

d) Find the time, to the nearest 0.01 seconds, that it would take the object fired into the medium with a velocity of 12 ms^{-1} to come to rest.

Question 11

The 5 kg mass is placed on a smooth plane inclined at an $\theta°$ to the horizontal as shown in the diagram below. The tension in the string is now T_2 newtons.



a) The tension force T_2 is labelled on the above diagram. Label all other forces acting on the 5 kg and 2 kg objects.

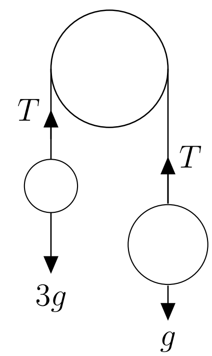
b) When $\theta = 30°$ the acceleration of the 2 kg mass is b ms^{-2} upwards. Find the value of b.

c) For what angle θ will the system be in equilibrium? Give your answer correct to 1 decimal place.

Question 12

The diagram shows particles of mass 1 kg and 3 kg connected by a light inextensible string passing over a smooth pulley. The tension in the string is T_1 newtons.

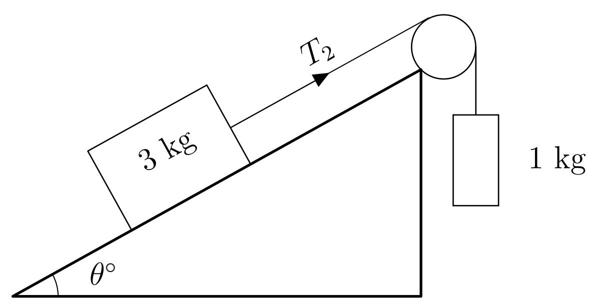
a) Let a ms^{-2} be the acceleration of the 3 kg mass downwards. Find the value of a.



b) Find the value of T_1.

The 3 kg mass is placed on a smooth plane inclined at an angle of $\theta°$ to the horizontal. The tension in the string is now T_2 newtons.

c) When $\theta = 30°$, the acceleration of the 1 kg mass upwards is b ms^{-2}. Find the value of b.



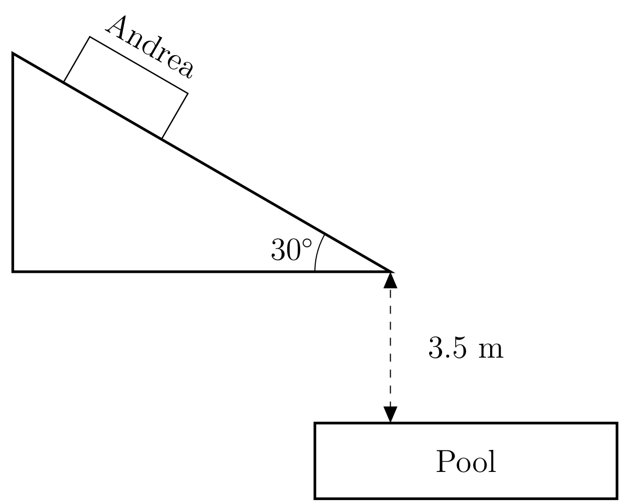
d) For what angle $\theta°$ will the 3 kg mass be at rest on the plane? Give your answer correct to 1 decimal place.

A rough inclined plane replaces the smooth inclined plane. The rough plane is inclined at an angle $\theta°$ to the horizontal. The coefficient of friction between the 3 kg mass and the rough plane is 0.2. The 3 kg mass will be at rest for $A \leq \theta \leq B$.

e) Find the values of A and B, giving your answer correct to 1 decimal place.

Question 13

WaterWorld has a water-slide into a deep pool. The 12.5 metre slide is inclined 30° to the horizontal and the bottom of the slide is 3.5 metres above the water surface of the pool. Andrea, who is of mass m kg, positions herself on the slide at the top and is supported by her friend Danny. When the applied force acting directly up the slide is $12g$ newtons, Andrea is on the verge of moving down the slide. The coefficient of friction between Andrea and the slide is $\dfrac{\sqrt{3}}{5}$.



a) On the diagram above, label all forces acting on Andrea while she is being held at the top of the slide by Danny.

b) Hence, calculate the value of m.

Suddenly, Danny releases his hold on Andrea, who moves from rest down the slide, reaching the end of the slide after travelling a distance of 12.5 metres.

c) Show that the acceleration of Andrea while she is on the slide is $\frac{g}{5}$ m/s^2.

d) Show that Andrea reaches the end of the slide with a speed of $7ms^{-1}$.

Take the bottom end of the slide as the origin, $\underset{\sim}{i}$ as the unit vector horizontally to the right and $\underset{\sim}{j}$ as the unit vector vertically up.

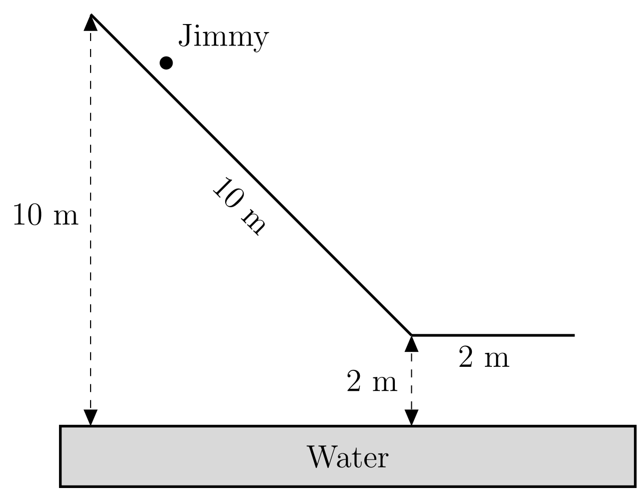
e) Find the velocity vector, $\underset{\sim}{v}(t)$ which represents the velocity of Andrea t seconds after she leaves the slide.

f) Calculate Andrea's speed 0.5 seconds after she reaches the bottom of the slide. Give your answer correct to 1 decimal place.

Question 14

A water-slide consists of a 10 metre inclined plane connected to a 2 metre horizontal plane. The top of the slide is 10 m above the surface of the water and the horizontal plane is 2 m above the surface of the water. Jimmy has a mass of 50 kg. He slides down the water-slide from the top, under the influence of gravity. The coefficient of friction between Jimmy and each surface of the water-slide is 0.4.

a) On the diagram below, mark in all the forces on Jimmy as he slides down the inclined plane.



b) Show that Jimmy accelerates down the incline plane at 5.488 m/s^2.

c) Find Jimmy's speed, in m/s correct to 2 decimal places, when he reaches the bottom of the inclined plane.

d) Show that Jimmy's speed at the end of the 2 metre horizontal plane is 9.70 m/s, correct to 2 decimal places.

e) When Jimmy leaves the end of the horizontal plane he moves under the influence of gravity and lands in the water.

i. If $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors horizontally to the right and vertically up. Show that a vector expression for $\underset{\sim}{r}(t)$, the position of Jimmy relative to the end of the horizontal plane t seconds after he leaves the water-slide is:

$$\underset{\sim}{r}(t) = 9.70t\underset{\sim}{i} - 4.9t^2\underset{\sim}{j}$$

ii. Hence, find the horizontal distance travelled by Jimmy after he leaves the water-slide, in metres correct to 1 decimal place.

Question 15

On the weekend, Dr. Rodgers, a maths teacher decides to go extreme snow skiing. The slope makes an angle of $22.5°$ with the horizontal ground. When Dr. Rodgers is at the top of the slope he is at a height of 75 metres. He starts his descent down the slope from rest.

a) Dr. Rodgers makes some quick mental calculations as he skis down the slope. First, he assumes there are no frictional forces or air resistance.

i. Find Dr. Rodgers' acceleration down the ski slope correct to 1 decimal place.

ii. Calculate Dr. Rodgers' speed at the bottom of the slope correct to 1 decimal place.

Dr. Rodgers arrives at the bottom of the slope and measures his speed to be a fast, but much more realistic speed of 23.4 m/s. He decides that there must have been significantly friction between his skis and the slope he didn't account for in his initial calculation. Dr. Rodger's mass is 80 kg.

b) Find, correct to 2 decimal places, the coefficient of friction between the slope and the skis.

At the bottom of the slope is a ramp that launches Dr. Rodgers into the air at an initial angle of 30° to the horizontal and a speed of 23.4 m/s respectively. Let t be the time in seconds after the launch and let the end of the launch ramp be the origin.
This leads to an acceleration of

$$\underset{\sim}{a}(t) = -\frac{kv_x}{80}\underset{\sim}{i} - \left(g + \frac{kv_y}{80}\right)\underset{\sim}{j}$$

where v_x is the horizontal speed at time t and v_y is the vertical speed at time t. On this particular day, $k = 20$.

c) Find the velocity vector in the form $\underset{\sim}{v}(t) = (a \times e^{-bt})\underset{\sim}{i} + (c \times e^{-bt} + d)\underset{\sim}{j}$. You should state the exact values of a, b, c and d.

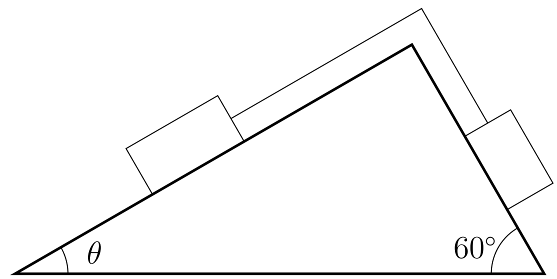
d) Hence, find the time Dr. Rodgers reaches his maximum height and his speed at this time. Give your answer correct to 1 decimal place.

e) Find the position vector in the form $\underset{\sim}{x}(t) = (p - p \times \alpha^t)\underset{\sim}{i} + (q - q \times \alpha^t + wt)\underset{\sim}{j}$. State the values of p, q and w correct to 1 decimal place and α correct to 4 decimal places.

f) If Dr. Rodgers lands at the same height he takes off, find the horizontal distance from the launch ramp he lands correct to 1 decimal place.

Question 16

Consider a 5 kg mass and a 3 kg mass connected by a light and inextensible string over a smooth pulley. The 5 kg and 3 kg masses lie on planes that are inclined at angles to the horizontal of $\theta°$ and $60°$ respectively. The two planes have coefficients of friction equal to $\mu = 0.1$ and $\mu = 0.2$ respectively. The masses are initially at rest.



a) Let $\theta = 45°$

i. Find, correct to 4 decimal places, the magnitude of the acceleration of the 5 kg mass and state whether it is sliding up or down the plane.

ii. Find the tension in the string. Give your answer in newtons and correct to 2 decimal places.

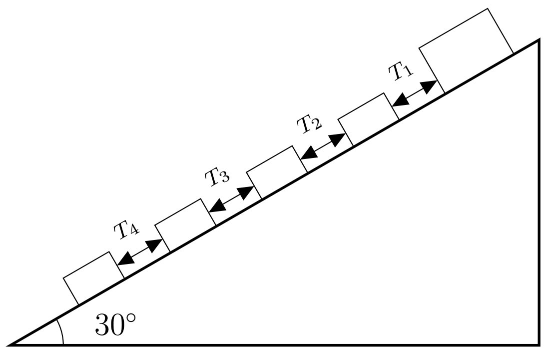
The string will break if the tension exceeds 32 newtons.

b) Find, correct to 2 decimal places, the angle θ at which the string will break?

Question 17

A toy train of mass 5 kg is attached by a light, inelastic string to four carriages. Each carriage has a mass of 1 kg. The train pulls the carriages up the frictionless ramp with a constant force of 20 newton. The ramp is inclined at an angle of 30° to the horizontal.

a) The tension force in each string has been marked, label all other forces acting on the train and carriages in the system.



b) The ratio $\dfrac{T_1}{T_4}$ is an integer. Find its value.

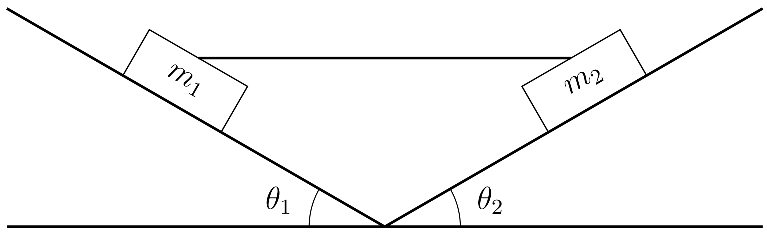
c) More carriages are attached at the rear using identical strings. All the carriages have a mass of 1 kg. If the strings will snap once the tension force becomes too great:

i. Which string will break first? Justify your answer.

ii. If more carriages of mass 1 kg are continually added, find the limiting value of T_1. Briefly explain whether or not the train continues to pull the carriages upwards as T_1 approaches the limiting value.

Question 18

Two particles of masses m_1 and m_2 are connected by a light horizontal rod and rests on two inclined planes. The planes are angled at θ_1 and θ_2 respectively to the horizontal as shown in the diagram below. The coefficient of friction between both masses and their respective planes is μ and the rod just prevents both masses from sliding down the plane.



a) On the diagram above, label all the forces acting on both masses.

b) The two forces exerted by the rod on both masses is called P_1 and P_2 respectively. Resolve P_1 and P_2 into component forces that are parallel and perpendicular to their respective planes.

c) Hence, show that $\dfrac{m_1}{m_2} = \dfrac{(\tan(\theta_2) - \mu)(1 + \mu\tan(\theta_1))}{(\tan(\theta_1) - \mu)(1 + \mu\tan(\theta_2))}$

d) Assume now that $\theta_2 = 2\theta_1$, and the rod is removed. The mass m_1 remains at rest, however the mass m_2 moves down the plane a distance of D metres in time T seconds. Show that $\theta_1 = \tan^{-1}\left(\dfrac{2D}{gT^2}\right)$