THE INSTITUTE FOR ENQUIRING MINDS

redefining
access
resetting
attitudes

enquiringminds.org
info@enquiringminds.org
+61 448 218652

# Specialist Maths Units 3/4

## Kinematics

## Practice Questions

# Short Answer Questions

## Question 1

A force $F = 3\sqrt{9 - v^2}$ is acting on a particle of mass 3kg moving in a straight line at time $t$ seconds. When $t = \frac{\pi}{3}$s, $v = \frac{3}{2}$m/s.

**a)** Find an expression for $v$ in terms of $t$.

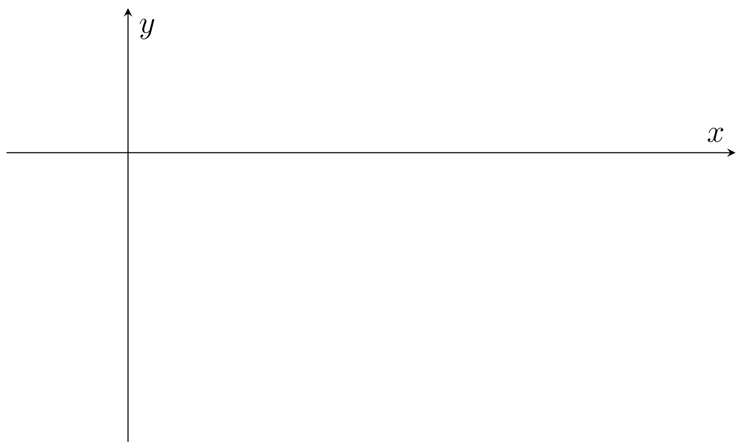
**b)** Find the time for the particle to come to rest for the first time.

## Question 2

An object is dropped from a balloon which is moving horizontally at an altitude of 200m. Due to air resistance the object experiences deceleration of $0.01v^2$, where $v$ m/s is the velocity at time $t$ seconds.

**a)** If $x$ metres is the altitude of the object $t$ seconds after it is dropped, write down an equation for the acceleration of the object and solve it to find an expression for the velocity in terms of $x$.

**b)** Sketch the graph of $v$ versus $x$.




**c)** Find correct to 2 decimal places, the object's velocity when its altitude is:

**i.** 180m

**ii.** 20m

**iii.** 0m

**d)** Find the terminal velocity and explain why it is not reached under this model.

## Question 3

Brett shoots an arrow from a bow straight into the air. 8 seconds later the arrow hits Brett in the shoulder.

**a)** How fast was the arrow initially fired? You can assume that the arrow begins and ends its flight at the same height.

**b)** What distance did the arrow travel before it hit Brett?

## Question 4

A parcel is dropped from a hot air balloon, which is 100m from the ground and ascending at 5m/s. Find the time taken for the parcel to reach the ground in terms of $g$, where $g$ is the acceleration due to gravity.

## Question 5

A toy rocket is projected from the ground vertically upwards with an initial velocity of 10 m/s and it rises with an acceleration of $a = -(g + v^2)$, where $g$ is acceleration due to gravity and $v$ is the velocity.

**a)** Express $x$ in terms of $v$ and $g$.

**b)** Hence, find the greatest height reached by the toy rocket in exact values in terms of $g$.

## Question 6

An object which is initially at rest at the origin begins moving with acceleration $a$ m/s$^2$ given by $a = \dfrac{1}{2+v}$ where $v$ m/s is the velocity of the object after $t$ seconds. The displacement of the object is $x$ m from the origin after $t$ seconds.

**a)** Find the exact position of the object when $v = 2$

**b)** Find an expression for $v$ in terms of $t$.

## Question 7

A mass has acceleration, $a$ m/s$^2$, given by $a = v^2 - 8$, where $v$ m/s is the velocity of the mass when it has displacement of $x$ m from the origin.

Find $v$ in terms of $x$ given $v = -3$ when $x = 1$.

## Question 8

A high diver jumps off a tower which is 44.1 metres above a deep tank of water. When the diver hits the water he is subject to an acceleration $a = -0.4(v + 0.6)^2$ m/s$^2$, $v \geq 0$ where $v$ is the velocity of water at any time $t$ seconds after impact with the water.

**a)** Find the velocity of the diver on impact with the water, assuming a constant acceleration of 9.8 m/s$^2$.

**b)** Find the velocity of the diver $v(t)$ in the water at time $t$.

**c)** How long after the diver leaves the tower does he come to rest in the tank?

**d)** How far below the surface of the water does he descend.

## Question 9

An object is dropped from rest so that the acceleration due to air resistance is $0.2v$ where $v$ is the speed of the object. The acceleration due to gravity is $g$ m/s$^2$.

$$\frac{dv}{dt} = g - 0.2v$$

**a)** Find $v$ in terms of $t$.

**b)** Find $\lim_{t\to\infty} v(t)$, that is, the maximum velocity.

**c)** Find the distance fallen after 5 seconds (to the nearest metre).

# Multiple Choice Questions

## Question 1

A body starts from rest with a uniform acceleration of 1.8 m/s$^2$. The time it will take for the body to travel 90m is:

**A.** 5s

**B.** $\sqrt{10}$s

**C.** 10s

**D.** $\sqrt{10}$

**E.** $10\sqrt{2}$s

## Question 2

A body moves in a straight line so that at time $t$ its acceleration is $a$ where $a = f(v)$. Given that $v = v_0$, when $t = t_0$, the time taken for velocity to change from $v_0$ to $v_1$ is given by.

**A.** $\displaystyle\int_{v_0}^{v_1} \frac{1}{f(v)}\ dv + t_0$

**B.** $\displaystyle\int_{v_0}^{v_1} (f(v) + t_0)\ dv$

**C.** $\displaystyle\int_{v_0}^{v_1} f(v)\ dv$

**D.** $f(v_1) + f(v_0) + t_0$

**E.** $\displaystyle\int_{v_0}^{v_1} f(v)\ dv + t_0$

## Question 3

A particle travelling in a straight line has velocity $v$ m/s at time $t$ seconds. Its acceleration is given by $\frac{dv}{dt} = -0.05(v^2 - 5)$. Its velocity is 50m/s initially and is reduced to 3m/s. Which one of the following is an expression for the time taken for this to occur.

**A.** $20 \int_5 0^3 \frac{1}{v^2 - 5}\ dv$

**B.** $20 \int_3^5 0 \frac{1}{v^2 - 5}\ dv$

**C.** $0.05 \int_3^5 0 \frac{1}{v^2 - 5}\ dv$

**D.** $0.05 \int_5 0^3 (v^2 - 5)\ dv$

**E.** $0.05 \int_5 0^3 (v^2 - 5)\ dt$

## Question 4

The acceleration of an object moving in a line at time $t$ seconds is given by $a = 10 - v$, where $v$ m/s is the velocity at time $t$ seconds. If the object starts from rest, then the time it takes to reach 5m/s, in seconds is approximately:

**A.** 0.69

**B.** 3.91

**C.** 2.71

**D.** 2.30

**E.** 3.00

## Question 5

A particle moves in a straight line with acceleration, in m/s$^2$, given by $a = 3 - x^2$ where $x$ is its displacement, in metres, from a fixed origin at $O$. If the particle is at rest at $O$, then the particle will also be at rest where.

**A.** $x = \sqrt{3}$ only

**B.** $x = -3$ only

**C.** $x = 3$ only

**D.** $x = \pm\sqrt{3}$ only

**E.** $x = \pm 3$ only

## Question 6

The acceleration, $a$ m/s$^2$, of a particle moving in a straight line is given by $a = -0.4\sqrt{225 - v^2}$, where $v$ is the velocity of the particle in m/s at time $t$ seconds. The initial velocity of the particle was 12m/s. The velocity of the particle in terms of $t$ is:

**A.** $v = -15\sin\left(\frac{2t}{5}\right) + 12$

**B.** $v = 15\sin\left(\frac{2t}{5}\right) + 12$

**C.** $v = -15\sin\left(\frac{2t}{5} + \sin^{-1}\left(\frac{4}{5}\right)\right)$

**D.** $v = 15\cos\left(\frac{2t}{5}\right) + 12$

**E.** $v = -15\cos\left(\frac{2t}{5} + \cos^{-1}\left(\frac{4}{5}\right)\right)$

## Question 7

A particle moving in a straight line has acceleration, $a$ ms$^{-2}$, given by $a = \sqrt{v+4}$, where $v$ is the velocity of the particle in ms$^{-1}$ at time $t$ seconds. Initially the velocity of the particle is $-3$ms$^{-1}$. The velocity of the particle at time $t$ is given by

**A.** $v = \left(\dfrac{3t}{2}\right)^{\frac{2}{3}} - 4$

**B.** $v = \left(\dfrac{3t+2}{2}\right)^{\frac{2}{3}} - 4$

**C.** $v = e^2 - 4$

**D.** $v = \dfrac{t^2-8}{2}$

**E.** $v = \dfrac{t^2-4t-12}{4}$

## Question 8

At time $t$ seconds, $t \geq 0$, the velocity $v$ m/s, of a particle moving in a straight line is given by $v = \cos(t) + \sqrt{3}\sin(t) - 1$. For what value of $t$ does the particle first attain its maximum speed of 3ms$^{-1}$.

**A.** $\dfrac{\pi}{6}$

**B.** $\dfrac{\pi}{3}$

**C.** $\dfrac{7\pi}{6}$

**D.** $\dfrac{4\pi}{3}$

**E.** The particle will never attain a speed of 3ms$^{-1}$.

## Question 9

A body is initially at the origin and moves in a straight line. Its velocity v m/s at time $t$ seconds is given by $v = 3\sin\left(\frac{\pi t}{12}\right) + 2$. After 26 seconds, the distance travelled by the body is closest to

**A.** 50 m

**B.** 53 m

**C.** 54 m

**D.** 60 m

**E.** 62 m

## Question 10

An object moves in a line so that $a = 8 - v$, where $v = 1$ when $x = 1$. At $v = 5$, the value of $x$ correct to two decimal places is:

**A.** 10.78

**B.** 2.78

**C.** 6.78

**D.** 3.78

**E.** 11.78

## Question 11

A particle moves in a straight line so that at time $t$, $t \geq 0$, its velocity $v$ and its displacement from a fixed point on the line $x$. If $\dfrac{dv}{dx} = \dfrac{1}{v}$, then the particle moves with

**A.** Constant acceleration and constant velocity

**B.** Constant acceleration and increasing velocity

**C.** Constant acceleration and decreasing velocity

**D.** Increasing acceleration and decreasing velocity

**E.** Decreasing acceleration and increasing velocity

## Question 12

A cricket ball is hit from ground level at an angle of $45°$ to the horizontal with an initial velocity of 20 m/s. The time $t$, in seconds for it to return to ground level is

**A.** $\dfrac{20\sqrt{2}}{g}$

**B.** $\dfrac{10\sqrt{2}}{g}$

**C.** $\dfrac{40\sqrt{2}}{g}$

**D.** $\dfrac{10}{g}$

**E.** $\dfrac{20}{g}$

## Question 13

Object $A$ is thrown down from the top of a cliff with the same speed and at the same time Object $B$ is thrown upwards from the bottom of the cliff. After 4 seconds Object $A$ and $B$ are both 50 metres above the ground. How long after Object $A$ reaches the ground does Object $B$ reach the bottom of the cliff (to one decimal place).

**A.** 6.6 s

**B.** 4.7 s

**C.** 2.5 s

**D.** 0.9 s

**E.** 1.9 s

# Extended Response Questions

## Question 1

A black bird flies past a stationary green bird sitting on a tree. When the black bird passes the green bird, at time $t = 0$ seconds, it has velocity of 20 m/s. The black bird flies in a straight line with an acceleration given by

$$\frac{dv}{dt} = -\frac{1}{50}v(v-2)$$

**a)** Using partial fractions and an appropriate integral, show that the black bird's velocity can be given by $v = \dfrac{20}{10 - 9e^{-\frac{t}{25}}}$.

**b)** Describe the long term behaviour of the black bird's velocity.

**c)** Calculate the distance the black bird will travel after 10 seconds, correct to 1 decimal place.

**d)** Calculate the time it will take for the black bird to travel 250m, correct to the nearest 0.1 sec.

Two seconds after the black bird passes the stationary green bird, the green bird decides to leisurely pursue the black bird. It travels in a straight line with a velocity given by $v = 5 + \sin(t)$, $t \in [2, 10]$.

**e)** Write down an expression which gives how far the black bird is ahead of the green bird when $t = 10$ seconds. Find this distance correct to the nearest 0.1 metre.

When $t = 10$ seconds, the green bird continues the pursuit with a constant velocity of 5 m/s.

**f)** At time $t = T_p$, the pursuing green bird catches the black bird. Write down an equation which when solved, will give the value of $T_p$.

**g)** Find $T_p$, correct to the nearest 0.1 second. Find the distance that the birds flew, in time $T_p$, correct to the nearest 0.1 metre.

## Question 2

A parachutist drops from a bridge and 'free falls' until he reaches a speed of 24.5 m/s.

**a)** **i.** Neglecting the effect of air resistance, find the time spent in 'free fall' to one decimal place.

**ii.** Find the distance travelled during this period, to two decimal places.

Having reached the speed of 24.5m/s, the parachute opens and the parachutist, whose mass is $m$ kg, is retarded by a variable force of $0.2mv^2$ N, where $v$ m/s is her velocity $t$'s after jumping from the bridge and $g$ m/s$^2$ is the magnitude of the acceleration due to gravity.

**b)** **i.** Taking the downwards direction as positive, write down the equation of motion of the parachutist during this second stage of descent and show that it simplifies to the differential equation $\dfrac{dv}{dt} = -0.2(v^2 - 5g)$.

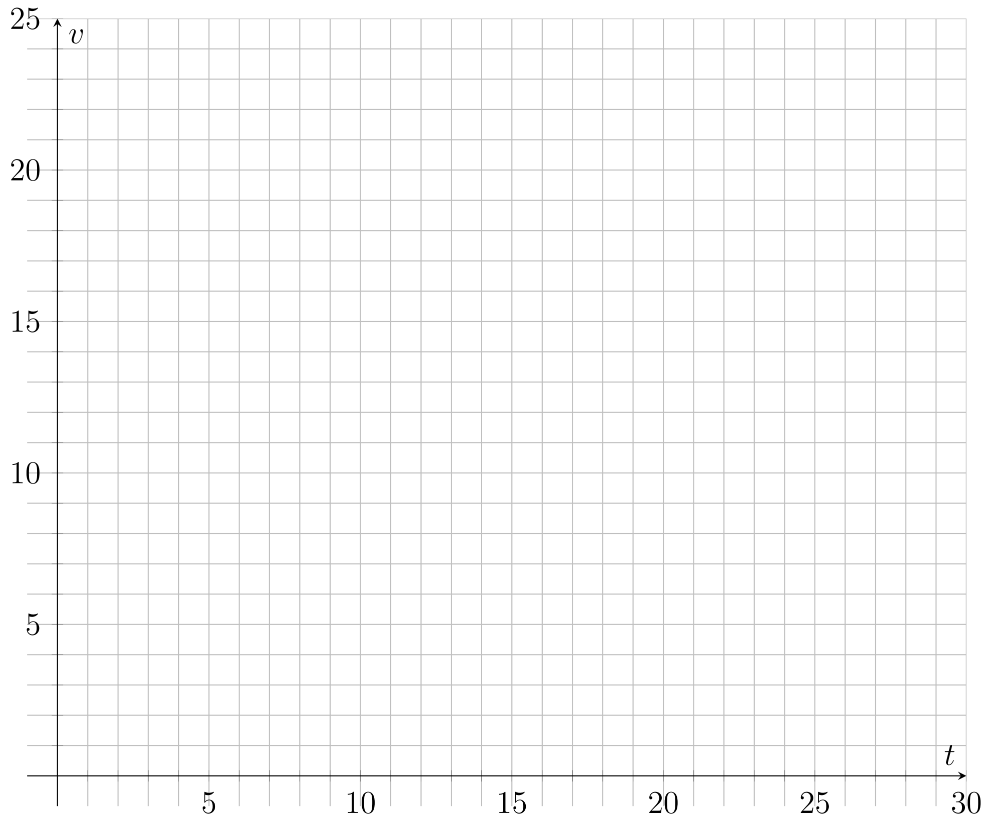
**ii.** Taking $g = 9.8\text{ms}^{-2}$, solve this differential equation to obtain in exact form, expressing your answer for $t$ in terms of $v$.

**iii.** Hence show, without using CAS, that $V = \dfrac{7(9e^{2.8t} + 5e^{7})}{9e^{2.8t} - 5e^{7}}$

The parachutist approaches a limiting velocity before she lands.

**c)** What is her limiting velocity? Show how you deduce your result.

**d)** Assuming that the parachutist lands 5 seconds after she drops from the bridge, sketch the velocity-time graph of her motion from the time that she drops from the bridge until an instant before she lands, showing all intercepts, endpoints and key features as co-ordinates to one decimal place.

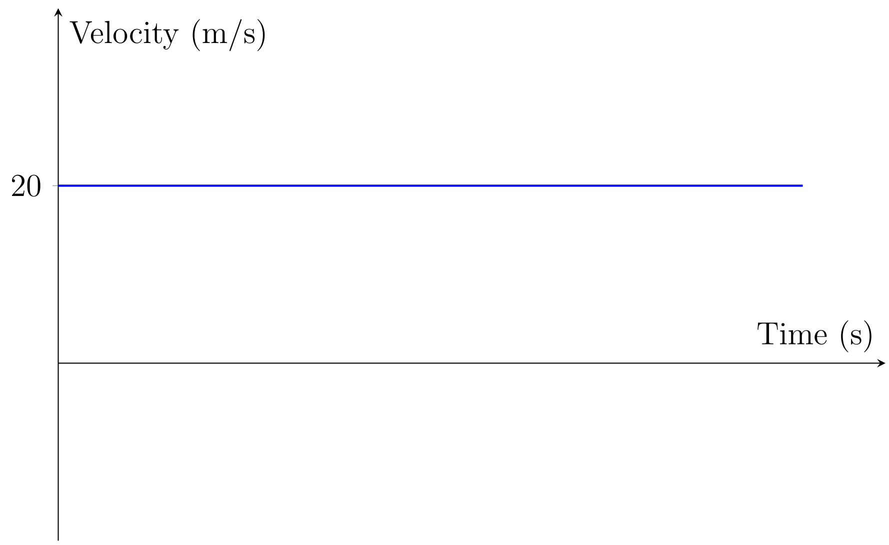


## Question 3

Dave owns a doughnut delivery business, he drives his van, "Dave's Doughnut Drifter" onto a straight stretch of freeway at time $t = 0$ and has a velocity in m/s at time $t$ seconds of $v(t) = 0.03t^2 + 15$.

**a)** What is Dave's entry speed onto the freeway?

Dave's competitor, Mike, drives his ute, "Mike's Monster Mover", on the same freeway and direction as Dave. Mike passes Dave the instant Dave gets on the freeway. Mike is travelling at a constant speed of 20m/s.
The velocity time graph for Dave's travel is shown below.




**b)** On the axes above, sketch the velocity-time graph for Dave.

**c)** Find, correct to two decimal places, the time when Dave catches up with Mike.
Further down the freeway, Dave spots what he thinks may be a speed camera and wets his pants. From that point on, his acceleration may be described by the equation:

$$\frac{dv}{dt} = -0.02(v^2 - 625), t \geq 0$$

**d)** Solve this equation to obtain $t$ in terms of $v$ given that at the start of this period of acceleration, $v = 35$.

**e)** Hence, show that $v = \dfrac{25(6e^t + 1)}{6e^t - 1}$/

**f)** If Dave were to maintain this pattern of deceleration, find his limiting velocity, explaining how you obtained your answer.

## Question 4

A ball bearing is released at $t = 0$ from rest at the surface ($x = 0$) into a swimming pool of depth 3.0m. The ball bearing is subject to two vertical forces, the weight force $mg$ and a drag force proportional to the velocity of the ball bearing. Take the acceleration due to gravity to be $g = 9.8\text{ms}^{-2}$.

**a)** Draw a force vector diagram to illustrate the two forces acting on the ball bearing as it moves vertically through the water.

**b)** Show that the acceleration $a$ can be written as $a = g - \frac{k}{m}v$, where $k$ is a constant of proportionality.

**c)** Find the velocity of the ball bearing $v(t)$.

**d)** Using the value $\dfrac{k}{m} = 2$, find $x(t)$ where $x$ is the distance below the surface of the water of the ball bearing at time $t$.

**e)** Use your equation $x(t)$ to find the time taken for the ball to hit the bottom of the pool. Give your answer accurate to two decimal places.

## Question 5

An object is projected vertically upwards from the ground with an initial speed of 30m/s. The object reaches a maximum height and then returns to the ground.

**a)** Assuming there is no air resistance, find:

**i.** The time taken for the object to reach its maximum height, correct to one decimal place.

**ii.** the maximum height reached by the object, correct to one decimal place.

Now assume that, due to air resistance, the acceleration in m/s$^2$ of the object as it moves upwards is given by $a = -g\left(1 + \frac{v^2}{100}\right)$.

**b)** **i.** Show that

$$t = \frac{10}{g}\left(\tan^{-1}(3) - \tan^{-1}\left(\frac{v}{10}\right)\right)$$

where $v$ m/s is the velocity at time $t$ seconds of the object as it moves upwards.

**ii.** Hence find the time, in seconds and correct to one decimal place, it takes the object to reach its maximum height.

**iii.** Show that while the object is moving upwards, its height in metres above the ground at time $t$ seconds is given by

$$x = \frac{50}{g} \log_e \left( \frac{1000}{100 + v^2} \right)$$

**iv.** Hence find, in metres and correct to one decimal place, the maximum height reached by the object.

## Question 6

While serving in tennis, a tennis ball is thrown vertically upwards with a speed of 4 m/s from a point 1.6 metres above the ground. While travelling upwards the tennis ball is subjected to a resistance force of $0.00294v^2$ N, where $v$ m/s is its speed at time $t$ seconds after it has been released. The mass of the tennis ball is 58.8 grams.

**a)** Show that the equation of motion of the tennis ball as it rises is given by

$$a = -\frac{196 + v^2}{20}$$

**b)** Using calculus, show that $v = 14\tan\left(\tan^{-1}\left(\frac{2}{7}\right) - \frac{7t}{10}\right)$

**c)** Hence find the time in seconds, correct to three decimal places when the tennis ball reaches its maximum height.

**d)** Write down a definite integral which gives the maximum height in metres, above ground level reached by the tennis ball.

**e)** Determine correct to three decimal places, the maximum height in metres, above ground level that the tennis ball reaches.

## Question 7

The differential equation $m\dfrac{dv}{dt} = mg - kv$ models the equation of motion of an object of mass $m$ kg falling vertically through the air, where $v$ ms$^{-1}$ is the speed of the object at time $t$ seconds, and $k$ is a positive real constant.

**a)** If an object dropped vertically from rest at time $t = 0$ is falling through the air, show that at time $t$ seconds, the speed of the ball is given by $v = \dfrac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right)$.

Yun, a skydiver, drops vertically from a hot air balloon so that $t = 0$, $v = 0$, and fall through the air for 10 seconds before opening her parachute. The combined mass of Yun and her skydiving equipment is 60kg.

**b)** If just before Yun opens the parachute her speed is 47.5 ms$^{-1}$, show that, before the parachute opens, the value of $k$, correct to the nearest integer is equal to 10. (Assume that the parachute opens instantly and that there is no upward thrust when it opens)

As $t \to \infty, v \to v_t$, where $v_t$ is the terminal speed of the skydiver.

**c)** Just before the parachute opened, what percentage of the terminal speed had Yun reached?
Assume that $k = 10$ and give the answer correct to the nearest integer.

**d)** Assume that $k = 10$, find the distance that Yun falls before the parachute is opened. Give the answer correct to the nearest metre.

**e)** The parachute opened successfully and Yun landed on the ground 2 minutes after dropping from the balloon, having reached a terminal speed of 6 ms$^{-1}$.

**i.** Show that after the parachute opens the value of $k$ is equal to 10g.

**ii.** Given that $t$ seconds after the parachute opens, $v = Ae^{-\frac{k}{m}t} + \dfrac{mg}{k}$, where $A$ is a real constant, show that the value of $A$ is 41.5.

**iii.** Hence find the height of the balloon above the ground when Yun dropped from the balloon. Give the answer correct to the nearest metre.