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Mathematical Methods Units 3/4

Probability

Practice Questions

– Discrete Random Variable

– Binomial Distribution

– Continuous Random Variable

– Normal Distribution

– Sampling and Estimation

Short Answer Questions

Question 1

At a carnival, it costs \$2 to buy a raffle ticket.

- One fifth of the tickets sold results in the buyer getting their money back.
- 65% of the tickets sold results in the buyer losing their \$2.
- 10% of tickets sold results in the buyer winning \$8 and getting their \$2 back.
- 5% of the tickets sold results in the buyer winning \$13 and getting their \$2 back.

a) Set up a probability distribution table, where W is the profit from buying a raffle ticket.

W				
$\Pr(W = w)$				

b) Hence, find the expected amount of money won or lost.

c) Do the people who run the carnival think this is a good game for the carnival? Explain with reasoning.

Question 2

The probability that Allan will phone Ben is 0.3, and the probability that Cameron will phone Ben is 0.2. If these events are independent, what is the probability that either Allan or Cameron will phone Ben.

Question 3

Suppose that a random variable, X, has a probability distribution:

x	0	1	2
$\Pr(X = x)$	0.5	a	b

a) Find the values of a and b if the mean of X is 0.7.

b) Find the variance of X.

Question 4

A variable X is known to be normally distributed with a mean, $\mu = 10$. If $\Pr(X > 8) = p$, write an expression for $\Pr(8 < X < 12)$ in terms of p.

Question 5

A spinning wheel at a fete is marked with the numbers 1 to 100. To play the game you pay \$2.00 and select one of these numbers. If the wheel lands on your number then you win \$50, otherwise you lose the \$2.00. If 1000 people play the game at the fete, find the expected profit for the organisers of the spinning wheel.

Question 6

Harry plays football each Saturday

- The probability that Harry plays well given that he slept well the previous night is 0.6.
- The probability that Harry plays well given that he didn't sleep well the previous night is 0.5.
- The probability that Harry sleeps well on a Friday night is 0.3.

a) What is the probability that on a certain Saturday Harry plays well.

b) What is the probability that Harry has slept well on a particular Friday night given that Harry has not played well the following Saturday?

Question 7

Find the value of a and b if $\mathrm{E}(X) = 7.1$.

x	2	4	6	8	10	12
$\Pr(X = x)$	0.1	0.2	a	b	0.1	0.15

Question 8

Consider a probability distribution defined by the rule $p(x) = \dfrac{2x+1}{35}, x \in \{1, 2, 3, 4, 5\}$

a) Confirm that this is a valid probability distribution.

b) Find $\mathrm{E}(X)$.

c) Find $\mathrm{E}(X^2)$.

For the probability distribution:

d) Find $\mathrm{E}(3X)$.

e) Find $\mathrm{Var}(X)$.

f) Find $\mathrm{SD}(X)$.

Question 9

Given the rule $p(x) = \dfrac{3x+5}{35}$ for $x \in \{0, 1, 2, ..., n\}$.
Find the value of n which makes $p(x)$ a probability distribution.

Question 10

In 2017 there were 50000 students doing Year 12 in Victoria. It is known that 10000 of them intended to take a GAP year in 2018.

a) What is the population proportion of students taking a GAP year in 2018?

Of the 50000 students sampled, 5 students are to be sampled

b) Complete the following table. Round the probability to 4 decimal places.
Estimate the probability of the sample proportion using binomial distribution.

Number of GAP Year students in sample	0	1	2	3	4	5
Proportion of GAP Year students in the sample, $\hat{p}$.						
$\Pr(\hat{P} = \hat{p})$						

c) Determine the probability that the proportion of GAP year students in the sample is more than 0.25.

Question 11

If a random variable X has a probability density function

$$\begin{cases} a + bx^2, 0 \leq x \leq 1 \\ 0, x < 0 \text{ or } x > 1 \end{cases}$$

and $\mathrm{E}(X) = \frac{2}{3}$, find a and b.

Question 12

a) Use calculus to find the derivative of $x^3 \log_e(x)$.

b) A continuous random variable, X, has a probability density function given by

$$f(x) = \begin{cases} -\frac{1}{4} x \log_e(x), & \text{if } 0 < x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Use calculus to find the exact mean value of X.

Question 13

A manufacturer of archery bows makes two types of bows, Quickdraw and Accuracy. 42% of bows made are Quickdraw and 15% of all bows are defective. If is found that 20% of defective bows are Quickdraw. Find the exact probability that if a non-defective bow is selected , that it is an Accuracy bow.

Multiple Choice Questions

Question 1

If a population proportion is believed to be 0.25 and samples of size 30 are chosen, what is the standard deviation of $\hat{p}$?

A. 0.00625

B. 5.625

C. 2.371

D. 0.079

E. 0.28

Question 2

If $\Pr(A) = 0.3,\ \Pr(B) = 0.5$ and the events A and B are mutually exclusive, then $\Pr(A \cup B) is:$

A. 0.65

B. 0.8

C. 0.15

D. 0.2

E. It is impossible to tell

Question 3

For the mutually exclusive events A and B that occur in a sample space, $\Pr(A) = 0.2$ and $\Pr(B) = 0.4$. Which one of the following statements is not true?

A. $\Pr(A \cup B) = 0.6$

B. $\Pr(A' \cap B') = 0.4$

C. $\Pr(A' \cap B) = 0.4$

D. $\Pr(A' \cup B') = 1$

E. $\Pr(A \cap B) = 0.08$

Question 4

A die has been altered so that the probability of throwing a 6 is 0.3. Alice rolls the dice 10 times. The probability that she obtains a 6 more than twice is closest to:

A. 0.2335

B. 0.3828

C. 0.2668

D. 0.6172

E. 0.8507

Question 5

Inside a container there are four black balls, five white balls and one red ball. Two balls are taken from the container, one after the other without replacement. The probability that no black balls are taken out is:

A. $\frac{5}{18}$

B. $\frac{2}{5}$

C. $\frac{3}{10}$

D. $\frac{3}{5}$

E. $\frac{1}{3}$

Question 6

The heights of the children in a queue for an amusement park ride are normally distributed with $\mu = 130$cm and $\sigma = 2.7$cm. 35% of the children are not allowed to go on the ride because they are too short.
The minimum acceptable height correct to the nearest centimetre is:

A. 125

B. 126

C. 127

D. 128

E. 129

Question 7

A jar contains seven white marbles, three green marbles and two blue marbles. Two marbles are drawn, with replacement, from the jar. The probability of drawing exactly one white marble is calculated by:

A. $\frac{7}{12} \times \frac{7}{12}$

B. $\left(\frac{7}{12} \times \frac{5}{11}\right) + \left(\frac{5}{12} \times \frac{7}{11}\right)$

C. $\frac{7}{12} \times \frac{5}{12}$

D. $2 \times \frac{7}{12} \times \frac{5}{12}$

E. $2\left(\frac{7}{12} + \frac{5}{12}\right)$

Question 8

Woolworths believes that 75% of people prefer their services to those of other companies. If they want their margin of error to be less than 5%, how many people do they need to survey to be 95% confident about their claim?

A. 73

B. 8

C. 289

D. 492

E. 528

Question 9

The continuous random variable X has a normal distribution with mean of 20 and standard deviation of 6. The continuous random variable Z has the standard normal distribution. The probability that Z is between -2 and 1 is equal to:

A. $\Pr(18 < x < 21)$

B. $\Pr(14 < X < 32)$

C. $\Pr(14 < X < 26)$

D. $\Pr(8 < X < 32)$

E. $\Pr(X > 14) + \Pr(X < 26)$

Question 10

If $\Pr(A) = \dfrac{2}{3}$ and $\Pr(A|B)$=$\Pr(B|A)$=$\dfrac{1}{2}$, then $\Pr(A' \cap B')$=

A. 0

B. $\dfrac{1}{6}$

C. $\dfrac{1}{5}$

D. $\dfrac{1}{4}$

E. $\dfrac{1}{3}$

Question 11

For the random variable X which has a probability density function:

$$f(x) = \begin{cases} 0, x < 0 \\ e^{-x}, x \geq 0 \end{cases}$$

the median is equal to:

A. e^{-2}

B. 1

C. 0.693

D. 1

E. 0.368

Question 12

The probability function of a binomial random variable X is shown. If the number of trials is 10 and p is the probability of a success on any trial, then the most likely value of p is:

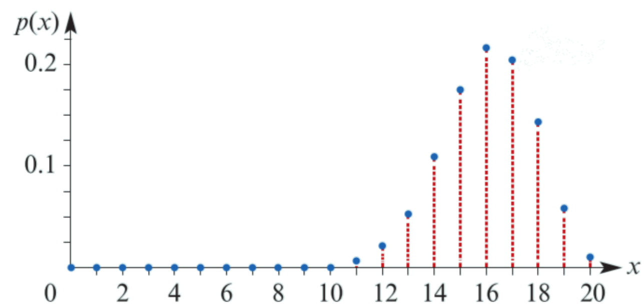
A. 0.3

B. 0.7

C. 0.1

D. 0.9

E. 0.1



Question 13

If random variable X has mean $\mu_x = 5$ and standard deviation $\sigma_x = 4$, and $Y = 2 - X$ then:

A. $\mu_y = 5$ and $\sigma_y = 4$

B. $\mu_y = -3$ and $\sigma_y = -2$

C. $\mu_y = -3$ and $\sigma_y = 4$

D. $\mu_y = -3$ and $\sigma_y = 2$

E. $\mu_y = 3$ and $\sigma_y = 2$

Question 14

John has three basketball commitments each week. He has two training sessions and 1 basketball match. The probability that he is late to exactly two of these in a week is given by:

$$^3C_2(0.4)^2(0.6)^1$$

The mean and variance of the number of times in a week that Jack is late for his basketball commitments are given respectively by:

A. $\mu = 0.8$ and $\sigma^2 = 0.48$

B. $\mu = 1.2$ and $\sigma^2 = 0.48$

C. $\mu = 1.2$ and $\sigma^2 = 0.72$

D. $\mu = 1.8$ and $\sigma^2 = 0.72$

E. $\mu = 3$ and $\sigma^2 = 4$

Question 15

Suppose there are two boxes. Box A contains 4 red balls and 6 black balls. Box B contains 3 red and 7 black balls. Suppose that a box is chosen at random, and then a ball at random is drawn from the box. The probability that Box A is chosen, given that a red ball is chosen is closest to:

A. 0.29

B. 0.35

C. 0.52

D. 0.48

E. 0.57

Question 16

The number of components in a batch that survive a given shock test us a random variable X which has binomial distribution with mean 15 and standard deviation 3. The probability of a component surviving the given shock test is:

A. 0.2

B. 0.4

C. 0.6

D. 0.8

E. 0.5

Question 17

The weight of packets of biscuits is normally distributed with a mean of 250g. If 92% of the packets have a weight of less than 260g, then the standard deviation, correct to one decimal place is:

A. 1.7

B. 1.4

C. 7.1

D. 9.1

E. 8.7

Question 18

The random variable X has a normal distribution with mean 20 and standard deviation 3. If Z has the standard normal distribution, then the probability that X is more than 26 is equal to:

A. $\Pr(Z > 3)$

B. $\Pr(Z < -3)$

C. $\Pr(Z < 2)$

D. $\Pr(Z < -2)$

E. $\Pr(Z > -3)$

For Q19 and Q20, the proportion of people who responded to a certain mail order catalogue is a continuous random variable X that has the probability density function.

$$f(x) = \begin{cases} \dfrac{2(x+2)}{5}, 0 < x < 1 \\ \\ 0, \ \text{elsewhere} \end{cases}$$

Question 19

The probability, correct to four decimal places, that more than one quarter but less than one half of the people contacted will respond to this mail order catalogue is:

A. 0.4500

B. 0.3875

C. 0.1000

D. 0.2375

E. 0.0896

Question 20

The mean of X, correct to four decimal places is:

A. 0.5333

B. 0.4000

C. 0.5000

D. 0.3667

E. 1.000

Question 21

The volume of liquid in a 1L bottle of soft drink is normally distributed with a mean of 0.95L and a standard deviation of 0.04L. The percentage of bottles that actually contain more than 1L of soft drink is closest to:

A. 11%

B. 50%

C. 8%

D. 89%

E. 16%

Question 22

The random variable X has normal distribution with mean 12.2 and standard deviation 1.4. If Z is the standard normal distribution, then the probability that X is greater than 15 is equal to:

A. $\Pr(Z < 2)$

B. $\Pr(Z > 2)$

C. $\Pr(Z > -2)$

D. $1 - \Pr(Z > 2)$

E. $1 - \Pr(Z < -2)$

Question 23

While on holiday at Philip Island, Jennifer and Barbara play a total of n games on mini-golf. The probability that Jennifer wins any game is 0.15. How many games of mini-golf must they play if the probability of Jennifer winning exactly two games is 0.2759?

A. 10

B. 20

C. 22

D. 54

E. 8

Question 24

A 10-sided die has four black sides and six red sides, and each side has an equal chance of showing. If the die is tossed four times, the probability of the same colour of showing at least three times is closest to:

A. 0.2800

B. 0.1792

C. 0.4992

D. 0.4752

E. 0.6544

Question 25

The discrete random variable X has the following probability distribution where $0 < a < 1$ and $0 < b < 1$.

x	1	2
$\Pr(X = x)$	a	b

$\mathrm{E}(X)$ is equal to:

A. $a + b$

B. $\dfrac{a+b}{2}$

C. $\dfrac{a+2b}{3}$

D. $3a + 1$

E. 1.5

Question 26

VB is sold in bottles labelled as containing 500mL. The amount of beer poured into each bottle in the bottle-filling process is normally distributed with a standard deviation of 2mL. Each bottle can hold no more than 510mL without spillage occurring. If only 5% of the bottles are over-filled so that spillage occurs, what is the mean amount of beer (correct to the nearest mL) poured into a bottle?

A. 504

B. 505

C. 506

D. 507

E. 508

Question 27

Andrew plays n games of squash. The probability that Andrew wins any games is 0.4, and no game can be a draw. For the probability Andrew wins all his games in a tournament to be less than 0.01, then the minimum number of games he would play is equal to:

A. 5

B. 6

C. 7

D. 8

E. 9

The probability of Roger beating Murray in a game of tennis is 0.56 and Roger and Murray decided to play a game of tennis every day for n days.

Question 28

The probability that over a week, Roger will win 4 games and Murray will win 3 is closest to:

A. 0.4962

B. 0.2932

C. 0.0588

D. 0.0084

E. 0.2304

Question 29

What is the fewest number of days on which they should play to ensure that the probability of Roger winning at least one game is more than 0.95?

A. 3

B. 4

C. 5

D. 6

E. 7

Question 30

The probability of winning a prize in a game is 0.12. The fewest number of games that must be played to ensure that the probability of winning at least two prizes is more than 0.50 is closest to:

A. 11

B. 12

C. 13

D. 14

E. 15

Question 31

When Tim goes padding, the time taken to complete his paddle, X minutes, is a continuous random variable with probability density function given by:

$$f(x) = \begin{cases} \dfrac{1}{1000}(x-20), 20 \le x \le 40 \\ \dfrac{1}{4000}(120-x), 40 \le x \le 120 \end{cases}$$

Tim paddles for n minutes 80% of the time. The value of n is:

A. 40

B. 60

C. 80

D. 100

E. 120

Question 32

X is a random variable with a probability function defined by:

$$f(x) = \begin{cases} 2\sin(2x), a \le x \le b \\ 0, \text{ elsewhere} \end{cases}$$

The values of a and b respectively are:

A. $0, \dfrac{\pi}{2}$

B. $0, \dfrac{\pi}{4}$

C. $-\dfrac{\pi}{2}, 0$

D. $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$

E. $-\dfrac{3\pi}{2}, -\dfrac{\pi}{2}$

Question 33

The random variable X has a normal distribution with mean 4.7 and a standard deviation 1.2. If Z is the standard normal distribution, then the probability that X is less than 3.5 is equal to:

A. $\Pr(Z < 1)$

B. $\Pr(Z > 1)$

C. $1 - \Pr(Z < -1)$

D. $\Pr(Z > -1)$

E. $\Pr(-1 < Z < 1)$

Question 34

If the continuous random variable X represents the time, in minutes, spent by shoppers at a Bunnings store on a Saturday, then X is normally distributed with a mean of 31 minutes and a standard deviation of 6 minutes.
The continuous random variable Z has the standard normal distribution.
$\Pr(-1 < Z < 2)$ is equal to:

A. $\Pr(19 < X < 37)$

B. $\Pr(30 < X < 30)$

C. $\Pr(25 < X < 37)$

D. $1 - \Pr(X > 25)$

E. $\Pr(X > 25) + \Pr(X < 43)$

Question 35

60% of people in a shopping centre are female. A random sample of 20 people is taken. The probability that his sample contains exactly twelve females is equal to:

A. $(0.4)^{12} \times (0.6)^8$

B. $(0.4)^8 \times (0.6)^{12}$

C. $^{20}C_{12}(0.4)^{12} \times (0.6)^8$

D. $^{20}C_{12}(0.4)^8 \times (0.6)^{12}$

E. $\dfrac{^{60}C_{12} \times ^{40}C_8}{^{100}C_{20}}$

Question 36

The weights of the members of a weightlifting squad are normally distributed with a mean of 70kg and a standard deviation of 9.7kg. 10% percent of these people are not allowed to enter a weight lifting event because their body weight is too low.
The minimum weight, in kg, of a person permitted to enter the weightlifting event is closest to:

A. 82.43

B. 80.01

C. 57.57

D. 56.86

E. 57.75

Question 37

A die is biased so that the chance of rolling a one is 0.3. Nancy rolls the die 15 times. Give that she rolled a one no more than 5 times, the probability that she rolled a one at least 2 time is:

A. 0.8511

B. 0.8242

C. 0.6864

D. 0.7540

E. 0.9316

Question 38

The height of a certain population is normally distributed with a mean of 175cm and a standard deviation of 8cm. A person is selected at random. If this person is greater than 170cm, the probability that they are also greater than 172cm is closest to:

A. 0.880

B. 0.737

C. 0.468

D. 0.646

E. 0.114

Question 39

If a student is late to school one day, there is a 20% chance that this student will be late on the next day. If a student arrives at school on time one day, there is a 12% chance that they will be late the next day. On a particular day it is found that 10% of students at school are late. The probability of students at the predicted to be late two days later is:

A. 0.8691

B. 0.1309

C. 0.1304

D. 0.8696

E. 0.1280

Question 40

A random variable X is such that its mean is 4 and its standard deviation is 3. $\mathrm{E}(X^2)$ is closest to:

A. 7

B. 5

C. 1

D. 25

E. 19

Question 41

The weight of a packaged orange cake is normally distributed with a mean of 500g. It the packaged orange cake is more than 5g underweight, it is not acceptable. It has been found that 4% of the cakes are unacceptable. The standard deviation of the weight of these cakes is:

A. 2.856

B. 5.000

C. 2.500

D. 3.856

E. None of the above

Question 42

A teacher determined that the top 20% of students in a class is given 'A' on the maths exam. If the distribution of scores on the examination is a random variable X with probability density function

$$f(x) = \frac{\pi}{200}\sin\left(\frac{\pi x}{100}\right), x \in 0 \leq x \leq 100$$

then the minimum score required to be awarded an 'A' is closest to:

A. 60

B. 70

C. 80

D. 90

E. 100

Question 43

If Z has the standard normal distribution and $\Pr(Z < k) = b$, where $k > 0$ and $0 < b < 1$, then $\Pr(|Z| > k)$ is equal to:

A. $2 - 2b$

B. $1 - b$

C. $2b - 1$

D. $2b - 2$

E. $b - 1$

Question 44

A manufacturer wishes to ensure that 98% of bolts that are produced from a manufacturing process have a diameter that lies within ± 0.05mm of the mean. For this to be so, the standard deviation of the process must be:

A. 0.0243 mm

B. 0.0215 mm

C. 0.0430 mm

D. 0.0304 mm

E. 0.0255 mm

Question 45

A and B are independent events of a sample space, where $\Pr(A) = a$ and $\Pr(B) = b$, where $a \in (0, 1)$ and $b \in (0, 1)$. If A' is the complement of A and B' is the complement of B, then $\Pr(A' \cup B')$ is equal to:

A. $1 - ab$

B. $1 + ab - a - b$

C. $ab - 1$

D. $a + b - ab$

E. $1 - 2a - 2b + ab$

Extended Response Questions

Question 1

The results of a SAC are found to be normally distributed with a mean of 62 and a variance of 100. Assuming a continuous distribution, and given that the pass mark is 50,

a) Find the probability that a student chosen at random failed the test.

b) If the top 10% of students are graded as having scored an 'A,' find the minimum score (to the nearest integer) necessary to be awarded an 'A.'

c) Find the probability that a student who is known to have passed, gets a score of at least 70.

d) If the top 30% of the students who failed are permitted to sit a supplementary test, find the minimum score possible for a student on the first test (to the nearest integer), who has failed but is allowed to sit the supplementary test.

Question 2

A group of six friends are going horse-riding. Each member of the group is randomly assigned a horse.

a) Seventy percent of the horses at the farm are brown.

i. What is the expected number of people to be randomly assigned a brown horse?

ii. Find the probability that two people are assigned brown horses. (Express your answer correct to 3 decimal places.)

iii. Find the probability that at least one person is randomly assigned a brown horse. (Express your answer correct to 3 decimal places.)

b) At the horse farm, the height of the horses is normally distributed with a mean of 150cm and a standard deviation of 8cm.

i. Find the probability, correct to 5 decimal places that a horse has a height greater than 160cm.

ii. What is the probability, correct to 5 decimal places, that everybody in the group is assigned a horse with a height greater than 160cm.

c) The group is assured that there will be at least two people in the group that will be assigned a horse which is taller than 160cm. What is the probability, correct to 4 decimal places, that exactly two people are assigned horses that are taller than 160cm.

d) A large number of Galloway ponies are kept at the horse farm. The height of these Galloway ponies are normally distributed with 15% of the ponies being less than 95cm tall and 20% of the ponies being greater than 107cm tall. Find the mean and standard deviation in centimetres of this distribution to 1 decimal place.

Question 3

Let X be a continuous random variable with the following probability density function:

$$f(x) = \begin{cases} 2\left(1 - \dfrac{1}{x^2}\right), 1 \leq x \leq 2 \\ 0, \text{ otherwise} \end{cases}$$

Find, correct to 3 decimal places:

a) a such that $\Pr(X \leq a) = 0.25$

b) b such that $\Pr(X \leq b) = 0.75$

c) the inter-quartile range of X.

Question 4

An examination for entrance into medical school is given to a group of prospective students. The results indicate an average of 312 with a standard deviation of 34.

a) What should the pass mark be if only the top 40% are to gain entrance?

b) What should the pass mark be if only the top 10% are to gain entrance?

Question 5

The Candlelite Company produces scented candles. These candles are sold in boxes of 25, of which 10 candles are jasmine-scented, 8 are musk-scented and 7 are sandalwood-scented. Joe, Kim and Li each buy a box of candles.

a) Kim opens her box of 25 candles and takes out 3 candles at random and lights them. Find, correct to 3 decimal places, the probability that Kim takes out 1 candle of each type of scent.

b) Li opens his box and takes out 4 jasmine-scented candles and 1 musk-scented candle and lights them. He then takes out another candle from the box at random. Find the probability that it will be a sandalwood-scented candle.

The Candlelite Company claims that 90% of all scented candles, regardless of scent, will burn for at least 30 hours.

c) Assuming that this claim is correct:

i. Find the probability, correct to 3 decimal places, that of the 25 scented candles in a box, 24 or more will burn at least 30 hours.

ii. What is the expected number of scented candles in a box of 25 that will burn for more than 30 hours?

The scented candles have a burning time which is normally distributed with a mean of 32 hours and a standard deviation of 1.5 hours.

d) Find, correct to 3 decimal places, the proportion of these scented candles that will burn for longer than 30 hours.

e) Li lights n candles. The probability that at least one of these n candles will burn for longer than 32 hours is 0.9375. Find the value of n.

The Candlelite Company can add a chemical to the candles to adjust μ, the mean burning time of the candles. The burning times of the candles, with the chemical added, is also normally distributed with a standard deviation of 1.5 hours.

f) Find the least value of μ, correct to 2 decimal places, so that 97% of the scented candles produced, with the chemical added, burn for 30 hours or more.

Question 6

The Balwyn Ball Company (BBC) makes tennis balls whose diameters are normally distributed with $\mu = 66$mm and $\sigma = 1$mm. The tennis balls are placed into cylindrical tins of six which are then packed into boxes of 24. A tennis ball fits into a cylindrical tin if the ball's diameter is less than 68mm.

a) What is the probability, to 4 decimal places, that a randomly selected ball fits into a tin.

The International Tennis Federation (ITF) defines the official diameter of a tennis ball to be between 65.4mm and 68.6mm.

b) What is the probability, correct to 4 decimal places, that a randomly selected ball made by the BBC satisfies the ITF's standards.

c) What is the probability, correct to 4 decimal places, that the diameter of a BBC tennis ball, which fits into a tin, satisfies the ITF's diameter requirement for a tennis ball?

4% of all balls manufactured by the BBC are considered flat as they do not bounce high enough.

d) What is the probability, correct to 4 decimal places, that a randomly selected tin of 6 selected balls contains, at most, one flat ball?

e) What is the probability, correct to 4 decimal places, that a box of 24 tins contains at least 20 tins with a maximum of one flat ball?

The BBC supplies tennis to the Balwyn Tennis Club. The Balwyn Tennis Club holds an annual tournament, which is a round-robin competition, for its members. In the round-robin competition, members are put into groups according to their age and each member then plays everyone else in that group.
Andy decides to enter the tournament and is put into a with 7 other players. The chance of Andy winning a match depends on how he has performed in his previous match. If Andy wins, the chance that he will win his next match is 0.78. If he loses, there is only a 0.36 chance that he will win his next match. Andy wins his first match.

f) What is the probability, correct to 3 decimal places, that Andy will wins his next three matches.

g) What is the probability, correct to 3 decimal places, that Andy will win two out of his next three matches.

Question 7

Kristina buys her bread from the Delighted Baker bakery. At the Delighted Baker the number X of slices in a loaf of thinly sliced bread is a discrete random variable with probability distribution as shown in the following table:

x	15	16	17	18
$\Pr(X = x)$	$\frac{1}{12}$	$\frac{1}{2-k}$	$\frac{5}{21}$	$\frac{1}{4} + k$

a. State the possible values of k.

At the Delighted Baker the number of Y of slices in a loaf of thickly sliced bread is a discrete random variable with probability distribution as shown in the following table:

x	14	15	16	17
$\Pr(X = x)$	0.21	0.38	0.23	0.18

b) Yolanda buys two loaves of thickly sliced bread from the Delighted Baker. Find the probability, correct to 4 decimal places, that:

i. each loaf has 14 slices.

ii. the total number of slices in both loaves is equal to 32.

iii. if the total number of slices in both loaves us equal to 32 then one of the loaves has 15 slices.

c) Luke buys seven loaves of thickly sliced bread from the Delighted Baker. Find the probability, correct to 4 decimal places.

i. at least three of the loaves have 16 slices.

ii. if at least two of the loaves have 15 slices then exactly four of the loaves have fifteen slices.

d) Ewan buys a number of loaves of thickly sliced bread from the Delighted Baker for his house-warming, Find the smallest number of loaves that Ewan could have bought if the probability that at least half of his loaves have less than 16 slices is greater than 0.86.

Question 8

A basic lottery game uses a machine to randomly select three balls, without replacement, from a barrel containing four white balls and six coloured balls.
Let X = the number of white balls selected per game.

a) Complete a tree diagram for drawing 3 balls from the barrel.

b) Hence, complete the probability distribution table below.

x	0	1	2	3
$\Pr(X = x)$	$\frac{1}{6}$	$\frac{1}{2}$		

c) Find:

i. $\Pr(X > 0)$

ii. $\mathrm{E}(X)$

iii. $\mathrm{Var}(X)$

This lottery game costs \$5 to enter and \$3 is paid out for every white ball selected.
Let Y= The profit/loss made by the player form one game.

d) Find, correct to the nearest cent:

i. $\mathrm{E}(Y)$

ii. $\mathrm{SD}(Y)$

Question 9

Another lottery game uses a machine to randomly select three balls, with replacement, from a barrel containing four white balls and six coloured balls.
Let X = the number of white balls selected per game.

a) Find, correct to 3 decimal places, the probability that:

i. Exactly one white ball is selected:

ii. More than one white ball is selected:

iii. Less than three white balls are selected given that at least one white ball is selected.

b) Find:

i. $\mathrm{E}(X)$

ii. $\mathrm{Var}(X)$

This lottery game costs \$$b$ to enter and \$$a$ is paid out for every white ball selected. Let Y = the profit/loss made by the player from one game.

c) Express Y in terms of X.

d) Hence find, in terms of a and b:

i. $\mathrm{E}(Y)$

ii. $\mathrm{Var}(Y)$

e) This lottery game is designed to make a 10% loss for the player in the long run.

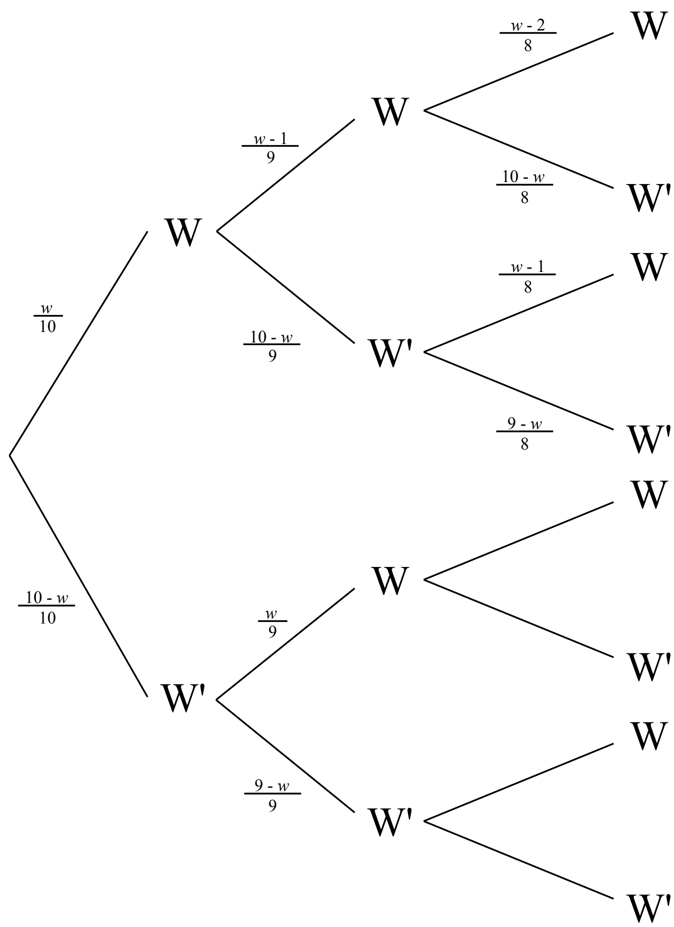
i. Show that $a = 0.75b$

ii. Hence, find the cost per game is $\mathrm{Var}(Y) = 14.58$

Question 10

Another lottery game allows the player to designate how many of ten balls are white. Three balls are selected at random, without replacement.
Let $w =$ the number of white balls designated by the player.
Let $X =$ the number of white balls selected per game.

a) Complete the tree diagram, expressing probabilities in terms of w.



b) Hence complete the probability distribution table

x	$\Pr(X = x)$
0	$\frac{1}{720}(10-w)(9-w)(8-w)$
1	$\frac{1}{240}w(10-w)(9-w)$
2	
3	

c) Find the value of w if more than one white ball is selected in 40% of games in which at least one white ball is selected.

d) This lottery game costs the player \$1 per ball they designate as a white ball. They must designate at least 3 balls to be white. The player receives \$1 for the first white ball selected and \$5 for each other white ball selected. Let $Y=$ the profit/loss made by the player from one game.

i. Find E(Y) in terms of w.

ii. This lottery game is designed to make a loss for the player in the long run. Find the maximum number of balls a player is allowed to designate as white balls.

iii. State the least strategically advantageous value(s) of w that a player could choose in the long run. Justify your answer.

e) This lottery game is modifies so that the three balls are selected with replacement. All other conditions remain the same.

i. State, in terms of w, the rule for the probability density function describing the distribution of X.

ii. Find the least number of white balls that should be designated to give at least a 90% chance of at least one white ball being selected in a game.

Question 11

The daily profit, P dollars, made by a lottery company is normally distributed with mean \$80000 and standard deviation \$25000. Each day is independent from any other.

a) Find the probability, correct to 4 decimal places, that the company makes:

i. Over \$100000 on a given day.

ii. Between \$60000 and \$90000 on a given day.

iii. Between \$60000 and \$90000 on five consecutive days.

iv. Between \$60000 and \$90000 on at least five out of seven consecutive days.

b) Find, correct to the nearest dollar:

i. The amount above which the company makes on 90% days.

ii. The interquartile range of P.

Question 12

Let Q dollars be the daily profit for another lottery company where $Q \sim N(\mu, \sigma^2)$. This company makes over \$100000 on 25% of days less than \$50000 on 10% of days. Find, correct to the nearest dollar, the mean and standard deviation of Q.

Question 13

A manufacturing company makes medals of two different standards, Regular and Superior. The quality of one medal is independent of the quality of any others it has manufactured. The probability that any one of the company's medals is Regular is 0.8.
The company wants to ensure that the probability that it produces at least two Superior medals in a day's production run is at least 0.9.
Find the minimum number of medals that the company would need to produce in a day to achieve this aim.

Question 14

Each morning Liv has either toast or cereal for breakfast. If she has toast one morning, then the probability that she has toast the next morning is 0.3. If she has cereal one mornings then the probability that she has toast the next morning is 0.6. Live has cereal for breakfast one Sunday morning.

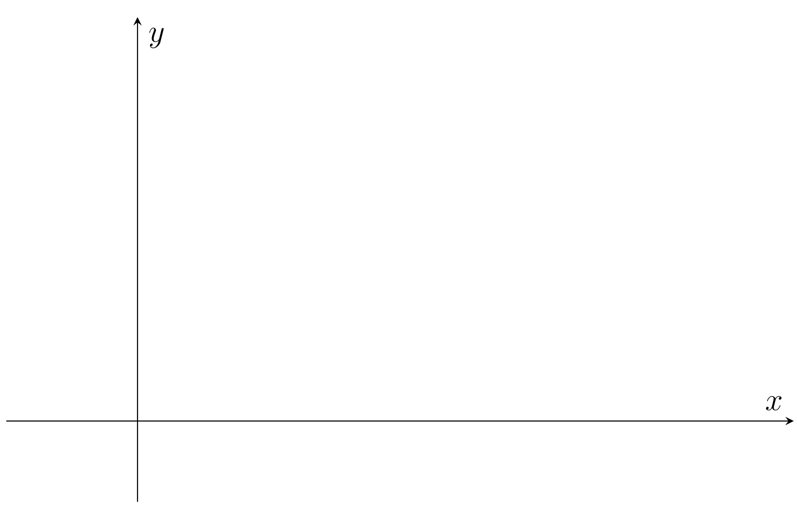
a) What is the probability that she will have cereal for the next two mornings?

b) What is the probability that she will have cereal just once over the next three mornings?

When Liv has cereal for breakfast, the mass, Xg, of cereal that she pours into her bowl is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{1000}(x-20), & 20 \leq x \leq 40 \\ \dfrac{1}{4000}(120-x), & 40 < x \leq 120 \end{cases}$$

c) Sketch the graph of $y = f(x)$ on the set of axes below.



d) One cup of Liv's breakfast cereal has a mass of 30g. Show that the probability of Liv having less than a cup of cereal for breakfast is 0.05.

e) Hence, what is the probability that Liv has less than 1 cup of cereal for breakfast on 2 of the next 10 mornings that she has cereal? Express your answer to 3 decimal places.

f) Find the probability that Liv has more than 2 cups of cereal given that she has nire than one. Express your answer as an exact value.

g) If $\Pr(X < n) = 0.8$, find the value of n.

Question 15

There are two types of cholesterol:
– High-density lipoprotein (HDL) - the 'good' cholesterol
– Low-density lipoprotein (LDL) - the 'bad' cholesterol
Cholesterol levels depend on a number of factors including genetic make-up, weight, level of physical activity and the types of food that you eat. Research shows that if you lower your blood LDL-cholesterol level you will lower your risk of heart disease and stroke.

a) The blood LDL-cholesterol level of an adult is a normally distributed random variable with a mean of 2.9 mmol/L and a standard deviation of 1 mmol/L.
Find the probability, correct to 4 decimal places, that the blood LDL-cholesterol level of an adult is:

i. Between 2.0 mmol/L and 3.1 mmol/L

ii. Less than 3.6 mmol/L

b) A blood LDL-cholesterol level greater than 4.1 mmol/L is considered high, a level less than a mmol/L is considered optimal and levels in between are considered acceptable.
The probability of a blood LDL-cholesterol level being optimal is equal to 0.3786.

i. Find the value of a. Give your answer in mmol/L correct to 1 decimal place.

ii. Find the probability that an adult with a high level of blood LDL-cholesterol will have a level greater than 4.3 mmol/L. Give your answer correct to 4 decimal places.

c) The probability that an adult has a blood LDL-cholesterol level between b mmol/L and 3.46 mmol/L is less than 0.5. Find, correct to 2 decimal places, the smallest possible value of b.

d) Find the probability, correct to 4 decimal places, that in a random group of nine adults.

i. Exactly five of the adults will have a blood LDL-cholesterol level between 2.0 mmol/L and 3.6 mmol/L.

ii. More than three of the adults will have a high blood LDL-cholesterol level.

e) The blood HDL-cholesterol level of an adult is normally distributed random variable. The probability of a blood HDL-cholesterol level being optimal is equal to 0.3085. The probability of a blood HDL-cholesterol level being poor is equal to 0.1587.
Find the mean and standard deviation of the blood HDl-cholesterol level of an adult. Give your answer in mmol/L and correct to 1 decimal place.

Question 16

At a sporting venue, a darts player has a 0.52 probability of hitting the bullseye and a 0.98 probability of hitting anywhere on the board (including the bullseye). Let X represent the number of bullseyes achieved out of 8 shots that the player takes.

a) Find $\Pr(3 \leq X \leq 8)$ correct to 4 decimal places.

b) State $\mathrm{SD}(X)$ correct to 4 decimal places.

c) What is the probability on any one shot that the player gets a bullseye given that he hits the board. Leave your answer correct to 4 decimal places.

d) Find the probability that the player gets at least 4 bullseyes in 8 shots if it is known that he gets a bullseye on the first shot. Give your answer correct to 4 decimal places.

A new competition on the same dartboard begins, the player wins if he gets 6 bullseyes.

e) Find the least number of shots he should have so that the probability that he gets at least 6 bullseyes is over 0.92.

Question 17

Coles supermarket claims that 90% of all the strawberries they sell are regular.
One of the customers collects a random sample of 51 strawberries after buying two large pockets and found that there are 47 regular strawberries in the sample.

a) Find the exact expected value and standard deviation, correct to 3 decimal places, of the sampling distribution of the proportion of regular strawberries in his sample.

b) Find, correct to 4 decimal places, the probability that at least 85% of the strawberries in his sample are regular. (Use binomial distribution)

c) Use an approximation of the normal distribution with the expected value and standard deviation in **part a.** to find the probability that at least 80% of the strawberries in his sample are regular. Give your answer correct to 3 decimal places.

d) **i.** Find the Z value that give a 98% confidence interval for the proportion of regular strawberries sold by Coles, correct to 4 decimal places.

ii. Hence, or otherwise, find the 98% confidence interval for the proportion of regular strawberries sold by Coles, correct to 3 decimal places.

Question 18

John Bond is a secret spy with the BCDF organisation. He needs to choose equipment before he goes on his next mission. He gets all his gear from the storeroom of the organisation. The storeroom holds several sizes of ammo boxes. The number of bullets in a huge box of ammo is a discrete random variable with the probability distribution shown below:

x	98	99	100	101	102	103
$\Pr(X = x)$	$\frac{1}{15}$	$\frac{1}{5}$	$2k$	$2k$	k	$\frac{1}{15}$

a) Show that the value of k is $\frac{2}{15}$

b) John randomly grabs 5 huge boxes of ammo.

i. What is the probability that exactly three of the boxes have 101 bullets in them? Give your answer correct to 3 decimal places.

ii. What is the probability that all the boxes contain more than 100 bullets, given that at least three of them contain more than 100 bullets. Give your answer correct to 3 decimal places.

c) What is the probability that one randomly selected box of ammo has either less than 100 bullets or more than 101 bullets.

d) What is the probability that if two huge boxes of ammo are randomly selected, then the total number of bullets will be more than 204?

e) Another spy from the organisation comes down to grab some ammo from the storeroom. Jane Bond randomly selects 8 huge ammo boxes. What is the probability that at least 6 of the boxes contain at least 100 bullets? Give your answer correct to 3 decimal places.

f) Jane is actually a double agent, and also works for the NCDF organisation. She does some research and finds that the mass of the bullets from NCDF are normally distributed. She finds that 35% of bullets have a mass of less than 2.9g and that 24% of bullets have a mass of more than 3.2g.
Find the mean and standard deviation of the mass of the bullets from NCDF, correct to 4 decimal places.

g) Jane wants to known more about the two different organisation's bullets, so she investigates the bullets at BCDF. SHe finds that they are also normally distributed. They have a mean of 3.012g and 26% of the bullets have a mass of more than 3.3g.

h) Jane classifies a bullet from either organisation heavier than 3.5g as heavy and any bullet with a mass of less than 2.5g as light.

i. What percentage of bullets from each organisation are classified as heavy, correct to 2 decimal places?

ii. What is the probability that a bullet from NCDF has a mass less than 2.2g give that it is already known that this bullet has been classified as light? Give your answer correct to 4 decimal places.

iii. What is the probability that a bullet from BCDF has a mass of more than 2.3g given that it is already classified as light? Give your answer correct to 4 decimal places.

i) Jane continues her research into the bullets at BCDF and NCDF. She is now interested in the diameters of all the bullets of each organisation. Jane considers any bullet with a diameter of between 0.99cm and 1.01cm to be regular. She takes a random sample of 220 bullets from BCDF and finds that 170 of the bullets are regular.

i. What is the exact value of $\hat{p}$ for this sample?

ii. Construct a 95% confidence interval for this sample, correct to 3 decimal places.

iii. If Jane was to repeat this sample 40 times, how many of the confidence intervals would she expect to contain the population proportion p of bullets that are regular?

j) Jane receives information from NCDF that after much investigation, they have concluded that 88% of all their bullets are regular. She decides to conduct her own research anyway, and again takes a random sample of 220 bullets.

i. Find, correct to 4 decimal places, the expected value and standard deviation of the sampling distribution of the proportion of regular bullets in her sample.

ii. What is the probability that more than 90% of the bullets that Jane has sampled are regular? Do not use normal approximation.

k) John has been feeling left out while Jane completes her research, he decides to do some research of his own. He goes back to the storeroom at BCDF and selects a smaller box of ammo that only has 20 bullets in it. He doesn't know yet, but 15 of the bullets in the box are regular.
John doesn't really think much of statistics and thinks that special military operations are more important, so he selects only 4 bullets from the box of ammo without replacement.

i. What are the possible values of $\hat{p}$ for John's sample?

ii. Fill in the probability distribution table which summarises the sampling distribution of the sample proportion of regular bullets when samples of size 4 are taken from the ammo box.

$\hat{p}$					
$\Pr(\hat{P} = \hat{p})$					

iii. What is the probability that in John's sample, more than 50% of the bullets are regular.

Question 19

John and Jane go on a cooperative mission together in the Amazon rainforest. They have a number of serious objectives, on day 4 of their mission, they need to split up. John needs to decipher an ancient code written on a large slab of rock near a river, and Jane needs to scale a rock wall nearby to gain a better view of their surroundings.

a) The probability that John succeeds is $\frac{2}{5}$, while the probability that Jane succeeds is p.

i. If John and Jane's success is independent of one another, and the probability that they both succeed is $\frac{4}{15}$, show that the value of p is $\frac{2}{3}$.

ii. If their success is not independent of each other, and the probability of either of them succeeding is $\frac{11}{15}$, show that the probability that they both succeed is given by $\frac{3p-1}{3}$.

iii. If we continue to assume that their success is not independent, and using $p = \frac{3}{5}$. Find the probability that John succeeds but Jane does not.

iv. Show that the probability that John succeeds, given that Jane succeeds is given by $\frac{3p-1}{3p}$.

On day 10 of their mission, John and Jane find themselves entering a previously unkown part of the jungle. Before they know it, they have fallen through a trap door onto a platform in a large well-lit underground cavern. While Jane manages to keep her feet and stay on the platform, John falls off the platform onto the floor of the cavern, which is writhing with a mass of hundreds of snakes.

The length of snakes in metres is a continuous random variable with the following probability density function.

$$f(x) = \begin{cases} ax^2(x-6), 0 \leq x \leq 6 \\ 0, \text{ otherwise} \end{cases}$$

b) Use calculus to show that the value of a is $-\dfrac{1}{108}$.

c) Find the probability that a randomly selected snake will have a length of more than 4.2m, correct to 4 decimal places.

d) Find the expected length of a randomly selected snake.

e) If Jane shoots 10 snakes in the pit. What is the probability that 6 of them were over 4m long.

f) Jane estimates that there are 3000 snakes in the pit. Approximately how many snakes would be between 1 and 2 metres in length. (to the nearest snake)

John and Jane manage to escape the snake pit and carry on with their mission. Part of their mission is to investigate the spread of an unknown disease amongst the Golden Stag Beetle. The disease is known to cause the beetles to become very large in size. The mass of the Golden Stag Beetle is known to be normally distributed with a mean of 2.5g and a standard deviation of 0.5g. Let X be the random variable that represents the mass of a stag beetle and let Z be the standard normal random variable.

g) First find d such that $\Pr(X > 2d) = \Pr(X < d)$

h) If $\Pr\left(Z > \frac{2}{3}\right) = 0.0668$, find q such that $\Pr\left(q < X < \frac{13}{4}\right) = 0.5461$, give your answer correct to 3 decimal places.

i) John and Jane discover that if a beetle is diseased then it has a mass greater than m grams. They know that the probability a beetle is diseased is 0.1932. Find the value of m correct to 4 decimal places.

j) Find the probability that a diseased beetle will have a mass greater than 3g. Give your answer correct to 4 decimal places.

k) Find the probability that in a random group of 20 beetles, that at least 5 of them will be diseased. Give your answer correct to 4 decimal places.

l) If Jane took a random sample of 165 from the population of beetles. What is the probability that less than 10% of the beetles would be diseased? Give your answer correct to 5 decimal places.

m) Jane wants to construct a 95% confidence interval for her sample, and she wants to have a margin of error of less than 5%. Given that the population proportion is approximately 0.1932. How many beetles should she include in her sample?

On the final day of their mission John and Jane decide to have some fun and test their marksmanship. Jane sets up a bullseye and John attempts to shoot it with his rifle. John knows he has a 40% chance of hitting the bullseye.

n) What is the smallest number of shots John must have to ensure that he has a probability of more than 0.95 of hitting the bullseye at least twice?

Question 20

A company that makes dog collars wants to have a stall at a Pets in the Park event. The organising committee of the event asked the company to submit a sample from their large supply of collars. The committee decided to test 7 of these collars.
– If more than 4 collars in this sample of 7 were poorly made, the company would be rejected.
– If 4 collars in this sample of 7 were poorly made, a second sample of 7 collars was tested. If any of the collars in this sample is unsatisfactory, the company's stall will be rejected, otherwise the company is approved.
The company know 30% of their collars of poorly made.

a) What is the probability that the company was rejected after the committee tested the first sample.

b) What is the probability that the company will be allowed to have a stall?

c) Find the expected number of collars to be tested.

Question 21

A bag contains 24 randomly distributed jellybeans of which 6 are black. Suppose two jellybeans are randomly selected from the bag without replacement.

a) Draw a tree diagram indicating all possible outcomes for black and non-black jellybeans. Mark the respective probabilities along each branch.

b) Find the probability that a black bean is drawn in the second selection given that a black bean is drawn in the first selection.

c) Find the probability that at least one non-black jellybean is selected.

The two jellybeans selected above are returned to the bag and mixed in with the others. Jellybeans are then again randomly drawn from the bag, but this time they are returned after being drawn. After 14 draws, the jellybean drawn is eaten.

d) What is the expected number of black jellybeans selected?

e) Find the probability that no more than 4 black jellybeans are selected? Give your answer correct to 2 decimal places.

f) If no more then 4 black jellybeans are selected, find the probability that at least 3 black jellybeans are selected.

g) Find the probability that a black jellybean is selected only on the last attempt.

Question 22

The number X of chocolate chips in a biscuit made at the Delighted Baker bakery is a discrete random variable with probability distribution as shown in the following table:

$\hat{p}$	1	2	3	4	5	6	7	8
$\Pr(X = x)$	k	$3k$	$5k$	$6k$	$5k$	$5k$	$3k$	$2k$

a) Show that $k = \dfrac{1}{30}$.

b) A customer buys two biscuit from the bakery. Find the exact probability that:

i. each biscuit have five chocolate chips in it.

ii. the total number of chocolate chips in both biscuits is equal to thirteen.

iii. If the total number of chocolate chips in both biscuits is equal to thirteen then one of the biscuits has six chocolate chips in it.

c) Another customer buys seven biscuits from the bakery. Find, correct to 4 decimal places, the probability that at least three biscuits have six chocolate chips in them.

d) The Delighted Baker decides to sell packets of the biscuit. Find the least number of biscuits that should be put in a packet so that:

i. the probability that at least one of the biscuits in a packet has more than five chocolate chips in it is greater than 0.9.

ii. the probability that at least three of the biscuits in a packet has more than five chocolate chips in it is greater than 0.8.

Crumbys is a new bakery that opened not far from The Delighted Baker. It was found that if a customer went to Crumbys one day then there was a 78% chance that the customer would go to The Delighted Baker the next day. However, if a customer went to The Delighted Baker then there was a 62% chance that the customer would go to Crumbys the next day.

e) It is equally likely that a certain customer will go to either bakery on Monday. Use a tree diagram to find the exact probability that:

i. the customer goes to the Delighted Baker on Tuesday.

ii. the customer went to the Delighted Baker on Monday if he goes to the Delighted Baker on Tuesday.

Question 23

A continuous random variable X has a probability distribution function given by:

$$g(x) = \begin{cases} \frac{x}{9}e^{-\frac{x}{3}}, 0 \leq x < \infty \\ \\ 0, \text{ otherwise} \end{cases}$$

a) Briefly explain why $g(x)$ is a probability distribution function.

b) Use calculus to find the derivative of $x^2 e^{-\frac{x}{3}}$.

c) Hence, use calculus and your answer to **part a.** to find the exact mean value of X.

d) Use calculus and algebra to find the exact mode of X.

e) Hence use calculus and algebra to show that the median value, m, of X is a solution to the equation:

$$(am + b)e^{-\frac{m}{3}} = c$$

where a, b and c are rational numbers.

Question 24

Steve buys all his strawberries in packets from the supermarket. The number of strawberries in a large packet is a discrete random variable with probability distribution as shown in the following table:

x	24	25	26	27
$\Pr(X = x)$	$\frac{1}{12}$	$\frac{1}{2-k}$	$\frac{5}{21}$	$\frac{1}{4} + k$

a. Find all possible values of k.

At the supermarket the number, Y, of strawberries in a small packet is a discrete random variable with probability distribution as shown in the following table:

y	14	15	16	17
$\Pr(Y = y)$	0.21	0.38	0.23	0.18

b) Steve buys two small packets of strawberries. Find the probability that:

i. each packet has 14 strawberries.

ii. the total number of strawberries in both packets is 32.

iii. If the total number of strawberries in both packets is equal to 32 then one of the packets has 15 strawberries.

c) On another occasion, Steve buys seven small packets of strawberries. Find the probability, correct to 4 decimal places that:

i. At least three of the packets have 16 strawberries.

ii. If at least two of the packets have 15 strawberries, then exactly four of the packets have 15 strawberries.

d) Eve, Steve's partner also buys strawberries from the same supermarket. She buys a small number of strawberries.

i. Find the smallest number of packets that Eve could have bought if the probability that at least half the packets have less than 16 strawberries is greater than 0.86.

ii. A small packet bought by Eve contains 15 strawberries, of those 12 are considered regular. From a random sample of 11 strawberries. Find the probability that more than 80% of the strawberries in the sample are regular.

e) The supermarket claims that 90% of all strawberries it sells are regular. Steve collects a random sample of 51 strawberries after buying two large packets.

i. Find, correct to 3 decimal places, the expected value and standard deviation of the sampling distribution of the proportion of regular strawberries in his sample.

ii. Find, correct to 4 decimal places, the probability that at least 85% of the strawberries in his sample are regular.

iii. Use the normal approximation to find the approximate probability that at least 80% of the strawberries in his sample are regular. Give your answer to 3 decimal places.

iv. Steve counts 47 regular strawberries in his sample. Construct a 98% confidence interval for the proportion of regular strawberries sold by the supermarket.

Question 25

Victor is a minor celebrity that has thousands of followers on Twitter. On any give day, the number, T, of tweets that Victor makes about his upcoming activities is a random variable with probability distribution given by:

t	0	1	2	3
$\Pr(T = t)$	0.1	0.3	0.4	0.2

a) Victor tweets on both Thursday and Friday. What is the probability that he makes a total of four tweets over these two days?

Victor wants to make a documentary about the *Stochastica* tribe that worships probability and statistics. In order to be allowed to film them, the tribe poses the following questions to Victor:

b) For the events A and B of a sample space. $\Pr(A) = \dfrac{1}{5}$ and $\Pr(B) = \dfrac{2}{3}$. Calculate $\Pr(A'|B)$ when:

i. A and B are independent.

ii. A and B are mutually exclusive.

iii. $\Pr(A \cup B) = \dfrac{3}{4}$.

c) Let the random variable X be normally distributed with mean 1.5 and standard deviation 0.4. Let Z be the standard normal random variable, such that $Z \sim N(0, 1)$.

i. Find a such that $\Pr(X < a) = \Pr\left(Z > \frac{a}{3}\right)$.

ii. Using the fact that, correct to 3 decimal places, $\Pr\left(Z > \frac{5}{3}\right) = 0.048$ and $\Pr\left(Z > \frac{5}{4}\right) = 0.106$, find k such that $\Pr(k < X < 2) = 0.846$.

iii. Find all possible values of b such that the function

$$f(x) = 4bx^3 + 3x^2 - b^2, x \in (0, 1]$$

is a probability density function.

Question 26

Tasmania Jones works in a factory making different types if cheese. A particular type of cheese ball made by the factory is ubiquitous. The ingredients for making each cheese ball is send to one of two machines, machine A and machine B.

a) The time, X minutes, taken to produce a cheese ball by machine A has the following probability density function.

$$f(x) = \begin{cases} \frac{3}{160}x^2, 0 \leq x \leq 4 \\ \frac{3}{40}(8-x), 4 \leq x \leq 8 \\ 0, \text{ elsewhere} \end{cases}$$

i. Find the median of X, correct to 3 decimal places.

ii. Find the mean of X.

iii. Find $\Pr(3 \leq X \leq 5)$

b) There is a 65% chance that it takes machine A less than a minutes to produce a cheese ball. Find the value of a, correct to 3 decimal places.

c) It can be shown that $\Pr(X \geq 5) = \dfrac{27}{80}$. Tasmania Jones randomly chooses a sample of 10 cheese balls produced by machine A. Find, correct to 4 decimal places, the probability that:

i. Exactly 7 out of the 10 cheese balls chosen take at least 5 minutes to produce.

ii. The first 3 cheese balls take at least 5 minutes to produce given that 7 of these 10 cheese balls take at least 5 minute to produce.

The time taken, Y minutes, to produce a cheese ball by machine B is normally distributed with a mean of 4 minutes and a standard deviation of 1 minute.

d) It is known that there is an equal chance that it takes both machine A and machine B less than b minutes to produce a cheese ball. Find the value of b, correct to 2 decimal places.

All of the cheese balls produced by machine A and machine B are stored in a large storage bin. There are twice as many cheese balls from machine A as from machine B in the bin.

e) A cheese ball is selected at random from the bin. It is found to have taken less than 5 minutes to produce. Find, correct to 4 decimal places, the probability that it was produced by machine A.

Question 27

Two-card is a very popular game of chance in the casinos of the Gold Coast. It is played in the following way: Six identical cards are placed face down on a table. Four of the cards are marked \$5, one is marked \$10 and the remaining card is marked \$20. A player chooses two cards at random without replacement and is paid an amount equal to the sum of the values of the two cards.

a) List the amounts of money that a player can be paid in a game.

b) Using a tree diagram or otherwise, calculate the exact probability of a player being paid :

i. \$10 in a single game

ii. \$30 in a single game.

c)

i. Calculate, correct to the nearest cent, the expected amount of money paid to a player in a single game.

ii. Hence or otherwise, determine correct to the nearest cent, how much money a player should pay to play the game if the game is fair.

d) Calculate the exact probability of \$15 being paid to the player if at least one of the cards chosen by the player is a \$5 card.

Tasmania Jones is holidaying in the Gold Coast and decides to play 12 consecutive games of Two-card.

e) Calculate, correct to 4 decimal places, the probability that Tasmania will be paid \$30 in at least three of these games.

f) Tasmania is paid \$30 in the very first game he plays. Calculate the exact probability that Tasmania is paid \$30 in only two of the other eleven games he plays.

g) Calculate the least number of games in a row that Tasmania would need to play to ensure that his probability of being paid \$30 in at least 5 of those games is more than 0.6.