REALITY IS ONLY A WORD

1. It Might be Weird but it Works!

Quantum mechanics[1] is the physics of the very small. The story I want to tell here will focus mainly on a peculiar feature of that world called **entanglement** and its implications for how we understand 'reality'. Erwin Schrödinger, one of the founding fathers of quantum theory, coined the term entanglement in 1935. He regarded it as "...*the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought." Certain aspects of quantum mechanics can appear weird, absurd even, yet it is an article of faith amongst physicists that quantum theory is the greatest human achievement of the 20th century.

It is both beautiful in its mathematical formulation and powerful in terms of its explanatory reach. Questions as diverse as why the sky is blue and the grass is green, to what makes rocks hard, to why the sun shines, all find their ultimate explanations in quantum mechanics. Its accuracy is unprecedented. There has never been a more finely tested physical theory[2]. Over the past hundred or so years it has been subjected to ever more ingenious and subtle experimental tests and its predictions have always been confirmed. Upon its validity rests the huge edifice of technology which defines modern life — it led directly to the invention of the transistor, a device which one way or another is behind all modern electronics. There would be no lasers without quantum mechanics nor would there be the nuclear magnetic resonance imaging technology so important in modern medicine. I could go on. In summary, quantum mechanics works!

So where's the weirdness? Well for one thing we're used to being able to visualise what's going on in our physical theories. But taking literally the quantum mechanical description of the atomic world, applying to it the visual grammar of our intuition, we find ourselves mired in a philosophical house of mirrors, encountering paradoxes at every turn. The problems arise from certain peculiar features of the quantum mechanical description of nature. There is the implication that an essential randomness is at work, at least in the sense that the answers to questions we can ask of the quantum world are in general and in principle random. This randomness seems not to have to do with a lack of complete knowledge of some deterministic mechanism at work 'behind the scenes', but is an inherent feature of how we are able to interface with the microworld. Another curious feature of the quantum formalism is the indirect way in which the current knowledge of a system, it's 'state', is encoded mathematically. This quantum mechanical 'state vector' is the central object of study and yet its status, beyond that of a calculational devise, is still hotly debated.

We'll explore these and other strange aspects of the quantum world and its formalism in what follows. I want to give as honest an account as possible of what makes quantum mechanics weird using essentially no mathematics but I also want to show you how physicists represent things in their actual work — I'm going to give you a genuine look at their symbolic language. So as not to obscure the

[1] I'll interchangeably use the terms quantum mechanics, quantum theory and quantum physics.

[2] In truth, quantum mechanics is regarded as nothing less that the correct framework for *all* of physics.

essence of the story I'm not going to talk, as a physicist typically would, in terms of a genuine atomic phenomena such as 'electron spin'. Instead, I'll keep things deliberately unspecific and talk simply of stuff (quantum stuff!) which can have just two properties, colour (blue or red) or shape (circle or square)[3]. Just because the presentation is cast in these less technical terms, rest assured that in no sense will I be dumbing down the treatment. I'm going to give it to you straight in a series of 'lessons'. Here comes the first!

2. Quantum mechanics Lesson 1 - Measurement and Preparation

Corresponding to the two properties of colour and shape, we have available to us two kinds of measurement, two experimental tests, which we can use to determine 'values' of the respective properties, blue or red in the case of colour, circle or square in the case of shape. If we test any piece of quantum stuff for colour then it will always produce the result red or blue. Likewise, conducting a shape measurement on a piece of quantum stuff we'll find either circle or square. Given a piece of quantum stuff of unknown origin we cannot say which of the two possible outcomes we'll obtain for a given measurement. However, if we perform two identical tests one after the other then we will always obtain identical results. For example, if we test a piece of quantum stuff for colour and find red and then immediately test for colour again, we are certain to obtain the result red. Similarly if we tested a piece of quantum stuff for shape and found square and then repeated the shape test on the same quantum stuff, then, with 100% certainty, the result would again be square. It is therefore said that by testing a piece of quantum stuff of unknown origin for colour or shape, by performing on it the respective measurement, we have **prepared** it in a known state of colour or shape respectively.

So far so sensible... Now let's do something a bit different. Suppose we prepare, as described, a large number of samples in states of known colour. That is, given a large collection of pieces of quantum stuff, all of unknown origin, we test each for colour and collect the red and blue samples separately into two subsets of known colour. Then the result of immediately repeating a colour test on a representative of either subset is known with certainty. But let's instead test, say, each of the blue subset for *shape*. We find that for about half of the samples the shape test gives the result circle with the other half of course giving the result square. Likewise for the red subset, there's roughly a 50:50 split between circle and square when a shape test is applied to each sample. In other words, given a piece of stuff prepared in a state of known colour a shape measurement is as likely to find that its circle as square. This is interesting. With the quantum stuff in a *known* state of colour I know the *probability* of finding it to be circle or square.

Let's now take a particular piece of quantum stuff which has been subjected first to a colour and then a shape test. Suppose the colour test revealed it to be red and the subsequent shape test result was square. Does this particular piece of quantum stuff then have both these attributes? Is it both square and red — a red square? Quite reasonably, you would think this must obviously be true and that if we now retest for colour we must surely confirm the earlier colour result, red. In fact this is *not* the case. There is now a 50% chance of it testing red and a 50% chance of it testing blue! The fact that it originally tested red for colour is *irrelevant*! The intervening shape measurement has caused us to lose all information about its colour. Likewise, if some quantum stuff was in a known state of shape and then subjected to a colour measurement. The colour measurement

[3]The idea of using colour and shape comes from a nice article by Frank Wilzek, "Entanglement Made Simple" available at `https://www.quantamagazine.org/20160428-entanglement-made-simple/`

would prepare it in a known state of colour, but the outcome of a subsequent shape measurement would be perfectly random. Shape and colour are an example of a pair of **complementary** properties. It is a fundamental fact of quantum mechanics that complementary properties cannot *simultaneously* be known. As a matter of principle, there is no test available to us that can simultaneously reveal both these properties. This is of course very different to what we're used to. What we could call 'classical' stuff, the objects of our direct physical experience, have properties which exist simultaneously and independently of whether we test for them. At those scales, red squares exist.

What it means for quantum stuff to have 'properties' is profoundly different from what it means for classical stuff to have properties. Physicists talk of the **state** of a system by which they mean all the information that's needed to completely describe the system. Since classical stuff can have both colour and shape its state could simply be described by a list of the values of its possible properties. For example,

$$(\text{🩸}, \bigcirc), \tag{1}$$

could describe some classical stuff having colour red and shape circle. The properties of quantum stuff may, as is the case for our colour and shape, be complimentary and so cannot be simultaneously known. The symbolic representation of quantum stuff is necessarily different. The sort of symbol used to represent a piece of quantum stuff might look something like,

$$|\Psi\rangle\,.$$

It's called a **state vector** and is very different from the simple list of properties we used to describe a piece of classical stuff. As we've seen, known shape implies no knowledge, no information, about colour and vice versa. In the quantum world we have seen that one and the same state may correspond both to the information that "the probability of the result of a colour test being red is 1" and to the information that "the probability of the result of a shape test being square is 1/2 and being circle is 1/2". So quantum stuff requires a very different kind of representation and then very specific rules to extract from that representation meaningful information. Consequently there is a deep and apparently necessary indirectness in the quantum description which one way or another accounts for the conceptual issues we face when trying to interpret what quantum theory is telling us about the world. Whilst we have a wonderfully successful mathematical formalism for extracting information from $|\Psi\rangle$ the status of exactly what $|\Psi\rangle$ *is*, is still being debated 100 years after the the theory's original formulation.

In our case, having tested some quantum stuff for colour and found red we have prepared the stuff in a state we could denote,

$$|\text{🩸}\rangle$$

or, if we found blue, the quantum stuff would be prepared in the state,

$$|\text{💧}\rangle\,.$$

Similarly if we performed a shape test on some quantum stuff we would prepare it in either the circle state,

$$|\bigcirc\rangle\,,$$

or the square state,

$$|\square\rangle\,.$$

The remarkable fact that, when we prepare some quantum stuff in a state of definite colour and then test for shape, there's a 50% chance of finding circle and 50% chance of finding square is encoded in the following symbolics.

$$\begin{aligned}
|{\color{blue}\blacklozenge}\rangle &= \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle + i\,|\square\rangle\right)\\
|{\color{red}\blacklozenge}\rangle &= \frac{1}{\sqrt{2}}\left(i\,|\bigcirc\rangle + |\square\rangle\right)\\
|\bigcirc\rangle &= \frac{1}{\sqrt{2}}\left(i\,|{\color{blue}\blacklozenge}\rangle + |{\color{red}\blacklozenge}\rangle\right)\\
|\square\rangle &= \frac{1}{\sqrt{2}}\left(|{\color{blue}\blacklozenge}\rangle + i\,|{\color{red}\blacklozenge}\rangle\right)
\end{aligned}$$

What these equations are 'saying' is that a state of known colour can be viewed as a **superposition** of shape states (the first two equations) and that likewise a state of known shape can be viewed as a superposition of known colour states (the third and fourth).[4] The important take away here is this notion of superposition. It means that two potential types of shape are simultaneously present 'in' the colour state and therefore sometimes when we test a state of known colour for shape we find circle and sometimes square. The numbers here ensure that, using the aforementioned rules for extracting information, the respective probabilities are exactly 1/2. It is very important to appreciate that if some quantum stuff is in such a superposition, say of circle and square states, it is *not* the same as it having *either* the shape property circle *or* square. To understand this, suppose we have a large number of pieces of quantum stuff in the known colour state red and another large number of pieces of quantum stuff which consist of a 50:50 mixture of circle and square pieces. It is certainly true that testing for shape we could not distinguish between these two collections — in both cases we'd estimate that half were square and half were circle. However, testing for colour does distinguish between them since the collection of pieces of quantum stuff which are superpositions are all in the known colour state red and so a colour test will have certainly have outcome red for each of those pieces whilst testing the 50:50 mixture of circle and square pieces will result in roughly a 50:50 split of red and blue outcomes. The reason being that each state of known shape is itself a superposition of red and blue states!

So, to reiterate, given some arbitrary piece of quantum stuff in some unknown state $|\Psi\rangle$, if we carry out a colour measurement and find the outcome red then we have prepared the piece of stuff in the state $|{\color{red}\blacklozenge}\rangle$. The outcome of another colour measurement on this piece is now clear — the mathematical representation of the state of the stuff is telling us it's red and testing for colour can only give this result! But if we test for shape then what's relevant is that this state has the quantum mechanical description as a superposition,

$$|{\color{red}\blacklozenge}\rangle = \frac{1}{\sqrt{2}}\left(i\,|\bigcirc\rangle + |\square\rangle\right).$$

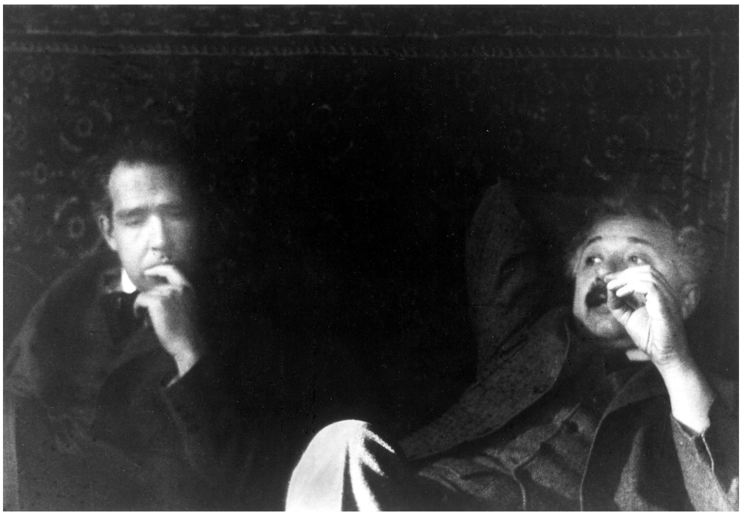
The outcome of a shape measurement is therefore random — both possibilities exist in the red state with equal probability. Perhaps we test for shape and find circle, then we have simply prepared the piece of quantum stuff in the state $|\bigcirc\rangle$. The information that it once had known colour is lost forever. It's worth pausing a moment here to reflect on just how different this behaviour is to that of our common experience. You might ask, but how can that be? I'm afraid I have no answer. This is how nature responds when we probe its microstructure in our macroscopic terms, that is, using our macroscopic laboratory equipment.

[4] Don't worry about the square root of 2 and the i's appearing here — they're just there because as promised I'm giving it to you straight, exactly as a physicist would write it.

Pre-quantum physics had been about establishing a mapping between the real world of our experience and a mathematical infrastructure, the physics, faithfully representing all aspects of that reality. Things at this scale have properties whether or not we test for them so the process of measurement plays no part in the classical formalism and we describe things directly as a list of the values of those properties, (💧, ◯). The quantum world is very different. We cannot separate properties, 'the world out there', from the tests we conduct. A test can take us from a minimal state of knowledge, $|\Psi\rangle$, to something like |💧⟩ *or* |◯⟩ but *not* both.

3. The Einstein-Bohr debate and the EPR Paper

Niels Bohr and Albert Einstein, giants of physics in the early 20th century, and central founding fathers of quantum theory, undoubtably had huge respect for one another. Yet they held radically different views of whether quantum mechanics deserved the status of a 'complete' physical theory and consequently the extent to which we should take seriously what it seems to tell us about nature.



Einstein in no way denied the efficiency and accuracy of quantum mechanics as a formalism for predicting the outcomes of atomic measurements, but he felt very deeply that it was somehow an expedient tool rather than a true rendering of a fundamental truth of nature. The idea that physical properties have in general no objective reality independent of the act of observation was anathema to him. He wrote in a letter to Max Born in 1944:

"*You believe in the God who plays dice, and I in complete law and order in a world which objectively exists...*"

In a letter to Schrödinger in 1950 he declared the necessity of a genuine physical theory to affirm

"*...reality as something independent of what is experimentally established.*"

The physicist John Wheeler, a towering figure of 20th century physics in his own right, who was in his early 20s at the height of the Bohr-Einstein debate and had close personal and professional contact with both protagonists, reflected[5] on their respective positions thus:

[5]In a 1985 BBC documentary, `https://www.youtube.com/watch?v=BFvJOZ51tmc&feature=share`

"Einstein admired Bohr and Bohr admired Einstein ... Einstein felt that reality exists, in effect, out there, independent of us, ... the position of Bohr was rather this, that reality is only a word and we have to learn what the right way is to use that word."

Anton Zeilinger[6] describes Einstein and Bohr as holding quite distinct views of the nature of physics. To Einstein, physics is about "what is", while for Bohr, physics is about "what we can say". Bohr's position acknowledges that our language, or, perhaps interchangeably, our visual intuition, is limited to the macroscopic world of our experience. When we want to learn and share information about quantum phenomena we must do so by imposing on the quantum world some macroscopic measurement apparatus which then defines what it is we can say about the quantum world. According to Wheeler,

"No elementary quantum phenomenon is a phenomenon until it's brought to a close by an irreversible act of amplification..."

It's the nature of that "irreversible act of amplification", the experimental setup, which determines what we can say about a physical system. As Pascual Jordan put it already in 1934, "*Observations not only disturb what has to be measured, they produce it... we ourselves produce the results of measurements.*". Regardless of Einstein's protestations this was the prevailing view of the broader physics community then, and still is today.

The Einstein-Bohr debate reached a climax in 1935 with the publication of the famous 'EPR paper'.

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Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

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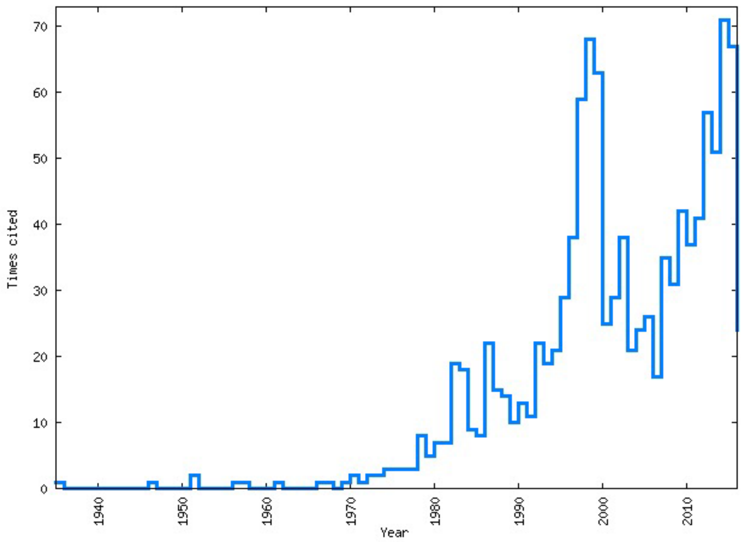
In it, Einstein, with two postdoctoral research associates, Boris Podolsky and Nathan Rosen, presented a challenge to the establishment's view of quantum theory of such cunning its repercussions still reverberate to this day. Indeed, it's interesting to note the physics community's rather unusual reaction to the paper as measured by citations per year since publication.

[6]Anton Zeilinger is the pre-eminent contemporary experimental quantum physicist. His views on the positions of Einstein and Bohr can be found, for example, in his Newton Medal lecture, `https://www.youtube.com/watch?v=7DiEl7msEZc`



Notice how it was essentially ignored for the first 30 or so years with interest only picking up at the beginning of the 70s. For a paper which has ended up being so important this is very unusual. The interest of the physics community would normally peak shortly after publication before levelling out in the long term as its contents become assimilated into the shared body of knowledge. Shortly, we'll see what happened around 1970 to trigger the surge of interest. Today it is by far Einstein's most cited paper — a remarkable, perhaps even perverse, fact given the scale of his contribution to modern physics.

So, to the eponymous question of Einstein, Podolsky and Rosen. Can a quantum mechanical description of physical reality be considered complete? Let's first be clear exactly what the question means. In the paper they define a complete theory as one in which "*every element of physical reality must have a counterpart in the physical theory.*" Their aim of course is to demonstrate that quantum theory is *not* complete by this criterion so we had better be able to identify both an "element of physical reality"[7] and whether or not it has a "counterpart in the physical theory". They provide the following test for recognising an element of physical reality.

> "*If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*"

In a physical system in which such an element exists, a "counterpart in the physical theory" then means that in the quantum mechanical formalism describing the system there is a precisely defined mathematical entity corresponding to that element.

All this should become clearer as we relate these notions to what we learnt in Lesson 1. It was described there how the act of measurement is not a passive process, it disturbs the system being observed. By testing some quantum stuff for colour we may prepare it in a particular state of colour. We would then know with certainty the result of a subsequent colour measurement. Thus, in EPR's terms, there exists an element of physical reality corresponding to the colour of quantum stuff so prepared and indeed in the mathematical formalism the state is described by a state vector bearing the label of the known colour, |🔵⟩ or |🔴⟩. The element of physical reality has its counterpart in the formalism. But recall that in this state we have *no* knowledge of the shape of our stuff and hence for quantum stuff

[7]There's something deliciously Pynchonesque about the author's initials also standing for the paper's key notion of an **E**lement of **P**hysical **R**eality!

in this state there is *no* element of physical reality corresponding to shape. To summon such an element of physical reality we would have to test for shape but that measurement would then cause the element of physical reality corresponding to colour to disappear.

As far as we've seen then, though complementarity imposes severe restrictions on how much can be known about a quantum system, the quantum theory does not fail the EPR criterion of completeness. The guile of Einstein and his colleagues was to exploit certain perfect correlations implied by the rules of quantum mechanics to apparently circumvent the either/or impasse imposed by complementarity, and allow the determination of a previously unknown property of a piece of quantum stuff *without* subjecting it to a direct measurement. These correlations arise from the remarkable feature of quantum mechanics called **entanglement**.

To get a feel for how EPR exploited perfect correlations consider the following simple (classical) example. Suppose each of a pair of gloves have been placed in two indistinguishable closed boxes. Two friends, Alice and Bob, then take a box each. Alice travels with her box to London, whilst Bob travels with his to Melbourne. Bob of course can't tell, without opening his box, whether he has a left-hand or right-hand glove and so can only guess at the handedness of Alice's glove. But the moment he opens his box in Melbourne and sees, say, a left-hand glove he knows immediately and with 100% certainty that the glove in Alice's box in London fits a right-hand. The result is already determined thanks to the perfect correlation between what Alice and Bob can observe when they open their respective boxes. In EPR terms, once Bob looks inside his box there exists an element of physical reality corresponding to the handedness of Alice's glove without Alice having to look inside her box. Of course to talk of 'elements of physical reality' in the context of gloves is just silly. We know fine well that gloves have a handedness whether we look at them or not and the perfect correlation is guaranteed since the two gloves in the example came from a single pair! But quantum stuff, as we've already learnt, can only be said to have a specific property if prepared through the appropriate act of measurement and the choice of type of measurement is up to us. In any case, crucially for the EPR attack on quantum mechanics, it turns out to be possible to cleave two pieces of quantum stuff from a single original piece in such away that they exhibit correlations similar to the two gloves. Such a pair are said to be **entangled**.

4. Quantum Mechanics Lesson 2 - Entanglement (i)

To describe the behaviour of entangled pieces of quantum stuff we enlist the help of our two friends, Alice and Bob. They are each stationed on different galaxies. Here on earth we produce thousands of entangled pairs of pieces of quantum stuff and for each one send one of the pair to Alice and the other to Bob. Upon receiving her piece of quantum stuff Alice, following our orders, flips a coin to decide wether to test for colour or shape. If she tests for colour she finds that there's a 50% chance of it being red and 50% chance of it being blue. Similarly, when testing for shape she finds the results to be totally random, there being an equal probability of it being square or circle. Exactly the same applies to Bob. Alone in their galaxies they thank us a bunch for doing nothing but sending them random bits of quantum stuff! Anyhow, they dutifully do their jobs and record the results of their tests for each of the thousands of pieces of quantum stuff they receive. Job done they return to earth and compare their results.

To their astonishment they find that whenever they both tested for the same property on each of an entangled pair they obtained the same results! For future reference, I'll summarise all possible results in the following table.

Alice(Colour)	Bob(Shape)	Alice(Shape)	Bob(colour)
blue	○	○	blue
blue	□	○	red
red	○	□	blue
red	□	□	red

Alice(Colour)	Bob(Colour)	Alice(Shape)	Bob(shape)
blue	blue	○	○
red	red	□	□

Notice that when Alice tests for colour and finds blue on one piece of a pair, Bob testing the other piece of the pair for shape can find either circle or square etc. There is no correlation. But, when they both happen to perform the same type of measurement on each piece of a pair the outcomes are always perfectly correlated. So in these cases its a bit like our gloves — Bob, having established the result of a particular type of measurement knows for sure the result Alice must obtain if she carries out the same type of test.

Imagine the following magic trick. We have a pair of dice and a pair of boxes. Each die behaves as you'd expect. You roll it time and time again and find the result to be random. Roughly 1/6th of the time you roll a 1, 1/6th of the time a 2 and so on. The two boxes are unremarkable but for the fact that they have two doors - one on the top and one at the front. Alice and Bob are now members of the audience who volunteer to assist the magician. Each is stationed at one of the boxes. The magician recruits a further member of the audience to roll the pair of dice and, blindfolded, place one in each of the two boxes. Alice is then instructed to announce the number she sees when opening her box using the door at the top while Bob announces the number he sees when opening the front door of his box. They repeat this time and time again sometimes switching so that Alice inspects her die from the front door and Bob his from the top but never using the *same* door. Just as the audience is becoming bored hearing Alice and Bob calling out random numbers with no apparent correlation between them the magician instructs Alice and Bob to start always inspecting their die using the *same* door. To the delight and consternation of the audience Alice and Bob now *always* call out the same number. I think you must agree, this would be a pretty neat magic trick. But this is essentially what is going on in the quantum phenomenon of entanglement! Please take a moment to reflect on how remarkable this is.

It is a prediction of quantum mechanics that we should be able to produce pairs of pieces of quantum stuff which, individually, behave with the signature quantum randomness, and yet as a pair exhibit the perfect correlations exhibited in the table above. Moreover, as laboratory experiments have verified, this quantum entanglement is a fact of nature.

5. The EPR argument continued

Armed with the quantum phenomenon of entanglement EPR now argue as follows. Since Alice and Bob are in different galaxies there is no way that Bob measuring the colour of his quantum stuff could have influenced the colour of Alice's quantum stuff or vice versa. As Einstein had established in his relativity theory there is a fundamental speed limit, light speed, which no causal influence can exceed. Their vast separation means that there is no chance of such an influence so the fact that whatever colour Bob finds, Alice, if she also measures colour, finds the same, must mean that her stuff had that colour or at least the 'instructions' to produce it already.

But Bob's decision of what to test for was based on a coin flip once he'd received his stuff. If the result of the coin flip had been different he would have tested his quantum stuff for shape, and that outcome would then also have been mirrored by a shape test of Alice on her piece. Since the pair of pieces had long since left earth when Bob flipped his coin, Alice's piece of stuff must also have been carrying shape instructions. In other words Alice's stuff must simultaneously have had colour and shape! At a stroke this apparently contradicts the complementarity of shape and colour and shines a bright spotlight on a fundamental *incompleteness* of the quantum mechanical description of nature.

Oh how EPR laugh! These are the results predicted by quantum mechanics and indeed the results observed in experiments. Quantum mechanics has caught itself in a trap of its own making. It does not admit simultaneous shape and colour elements of physical reality — in its formalism a state is not represented as a list such as, (🩸, ◯), a quantum state can either have known colour, $|$🩸$\rangle$, or known shape, $|$◯$\rangle$, not both. Yet we appear to have used the mechanics of this symbolism, quantum mechanics, to engineer a situation in which some quantum stuff demonstrably *does* have both properties! Rejecting, as EPR do, that some kind of strange instantaneous action-at-a-distance links the two pieces of quantum stuff so as to magically orchestrate the perfect correlation, there must be some hidden instructions being carried with the pieces of quantum stuff from when they were together on earth. These hidden instructions, typically called **hidden variables**, are elements of physical reality which are clearly missing counterparts in the quantum formalism and therefore, argue EPR, quantum mechanics must be regarded as incomplete.

The physics community turned, as they had done so often throughout the great Bohr-Einstein debate, to Bohr for the resolution of the apparent paradox. To save, once more, quantum mechanics from Einstein's latest insurgence! Bohr's infamous response honed in on the fact that Einstein was making use of what's technically called 'counterfactual reasoning'. When EPR argue that Alice's stuff must have both a predetermined colour and shape they are making use of a test that was never performed. If Bob tests for colour then Alice's stuff surely will be found to have the same colour. But Bob can't simultaneously test for colour and shape. Therefore, Bohr argued, it is not admissible to argue that because Bob *could have* performed a different test, namely a shape test, that Alice's stuff also has determined shape. In his inimitable way, Bohr expressed this as the fact that Bob's test, whatever it is, has "*an influence on the very conditions which define the possible types of predictions regarding the future behaviour of...*" Alice's stuff. It's hard to believe that this really did sate the community's desire for a satisfactory resolution but quantum mechanics was by then already proving itself an extraordinarily successful framework for understanding the microscopic foundations of the stuff of nature, and the majority of the community simply returned to the job of exploring its further implications and applications.

But of course there *were* lingering doubts. Einstein himself soon distanced himself from the emphasis in the EPR paper on the *simultaneous* reality of colour and shape properties, from the refutation of Bohr's precious complementarity principle. Instead, his discontent focussed on the simple lack, as he saw it, of a satisfactory explanation within the quantum formalism of the perfect correlations observed. As Einstein put it in a 1947 letter to Max Born,

> "*I cannot seriously believe (in quantum theory) because it cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance.*"

His concern now centred on the issue that if quantum mechanics is to be believed, in the sense that there are no hidden variables at work predetermining the measurement outcomes of Alice and Bob, then how can it be that the measurement outcome of Bob 'determines' that of Alice. Returning to our magic trick, how can it be that Bob inspecting his die from the top door and seeing a 6 *guarantees* that if Alice also inspects her die through the top door she must also see a 6 even though, working individually, choosing between the two doors at will they observe nothing but the expected randomness of a rolled die? To be fair to Einstein, I think its quite understandable that he chose not to believe that nature could be so strange! In another letter to Born, in 1948, he put it thus.

"*Those physicists who regard the descriptive methods of quantum mechanics as definitive in principal would...drop the requirement for the independent existence of the physical reality present in different parts of space; they would be justified in pointing out that the quantum theory nowhere makes explicit use of this requirement. I admit this, but would point out: when I consider the physical phenomena known to me, and especially those which are being so successfully encompassed by quantum mechanics, I still cannot find any fact anywhere which would make it appear likely that (the) requirement will have to be abandoned. I am therefore inclined to believe that the description of quantum mechanics...has to be regarded as an incomplete and indirect description of reality...*"

His point here is that the simple fact of Bob, on his distant galaxy, deciding to test his piece of quantum stuff for colour surely cannot "through some spooky action at a distance" determine the outcome of a colour test made by Alice on her piece of quantum stuff in a distant galaxy. Thus, though the quantum formalism doesn't seem to *need* to require preexisting properties, some form of instruction sets, to give rise to these properties, they *must* be there and so something is missing! Einstein's reasoning seems reasonable and yet the hard truth was that quantum mechanics seemed to be doing just fine without the so called 'hidden variables' (instruction sets) and to many Einstein appeared to be wasting his time on a purely philosophical question. As Wolfgang Pauli put it, somewhat uncharitably(!), in a letter to Born in 1954:

"*One should no more rack one's brain about the problem of whether something one cannot know anything about exists all the same, than about the ancient question of how many angels are able to sit on the point of a needle. But it seems to me that Einsteins questions are ultimately always of this kind.*"

6. An Explanation of the Quantum Correlations?

It's actually rather easy to see how a satisfactory 'hidden variable' explanation *can* account for the observed correlations. We suppose that somehow the process of producing these entangled pairs produces pairs of pieces of quantum stuff that are in fact carrying instructions of how to respond to colour or shape measurements. In a single run, then, it might be that both pieces of the pair were carrying instructions for blue circle. Of course *we* don't have access to these instructions, they're *hidden*, so we can't say before Alice and Bob carry out measurements what the outcomes will be. But *nature* 'knows' that if Alice or Bob carry out a shape measurement they'll both observe the outcome circle and if they test for colour they'll find blue. If, then, in the process of producing thousands of such entangled pairs 25% of the time both pieces of quantum stuff carry with them those instructions for blue circle, 25% of the time for red circle, 25% of the time for blue square and 25% of the time for red square then this would reproduce the findings of Alice and Bob. Individually, 50% of the time when Alice or Bob test for colour they'd find blue and 50% of the time they'd find red. Likewise, when testing for shape they'd find circle

50% of the time and square 50% of the time. Thus, as required, their individual results would appear perfectly random. But, on those occasions that they both tested for colour or shape on two pieces of entangled quantum stuff, that is, they both tested for the same property on a single run of the experiment, they'd find that they always agreed and that 25% of the time they'd both observe red, 25% of the time blue, 25% of the time circle and 25% of the time square. All of this in perfect agreement with the experimental results as summarised in the table above.

So perhaps Einstein was right? Perhaps that's how it works and the quantum formalism is somehow missing these hidden variables working in the background. But how could we ever test for their existence? Remarkably, that question lay unanswered for about 30 years, until in 1964 John Bell made a startling discovery. But before we get to that let's round out our discussion of entanglement by seeing, at least symbolically, how quantum mechanics achieves what we've managed here with our hidden variables.

7. Quantum Mechanics Lesson 3 - Entanglement (ii)

The pair of pieces of quantum stuff are entangled because they originate from a single piece in a particular way such that their state is represented by the following expression.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|{\color{blue}\blacklozenge}\rangle_A\,|{\color{blue}\blacklozenge}\rangle_B - |{\color{red}\blacklozenge}\rangle_A\,|{\color{red}\blacklozenge}\rangle_B\right)$$

The A and B subscripts here denote Alice and Bob's piece of quantum stuff. This is the mathematical representation of the state of the pair of distantly separated pieces. Notice that we can't say that Alice certainly has a blue or red piece of stuff. The overall description is a sum of two possibilities. It's a superposition. It's like what we saw before with a state of definite colour being described as a superposition of the two possible shape states.

Now suppose Bob tests his piece of quantum stuff for colour and finds blue. This new information eliminates the red possibility from the description of the state leaving us with the pair described by the state $|{\color{blue}\blacklozenge}\rangle_A\,|{\color{blue}\blacklozenge}\rangle_B$ and we see then that Alice's piece of stuff must also have blue colour. If she tests for colour she is *sure* to find blue. On the other hand, this post Bob measurement state can also be written as

$$|{\color{blue}\blacklozenge}\rangle_A\,|{\color{blue}\blacklozenge}\rangle_B = \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle_A + i\,|\square\rangle_A\right)|{\color{blue}\blacklozenge}\rangle_B$$

which shows us that if Alice instead tests for shape then the result is entirely random. There's a 50% chance she'll find circle and 50% chance she'll find square. When they don't test for the same property there is no correlation.

We can also rewrite the original state in terms of shape states as follows (this is simply a matter of plugging in the expressions we saw earlier relating known colour state to superpositions of shape states).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle_A\,|\bigcirc\rangle_B - |\square\rangle_A\,|\square\rangle_B\right)$$

In this form we see, for example, that if Bob tests for shape and finds square then the updated description of the entangled pair is described by the state $|\square\rangle_A\,|\square\rangle_B$ and so it is clear that if Alice measures shape she too will find square.

8. John Bell's contribution

In the discussion of the EPR argument I focussed on Einstein's belief that there must exist some hidden variables accounting for the perfect correlations. This is often described as the assumption of **realism**. Einstein's insistence that physics be about 'what is'. The idea that things have properties whether or not we test for those properties. To Einstein and his coworkers this requirement was the inevitable conclusion assuming the quantum results since they took it as self-evident that there could be no "spooky" communication between the two pieces of quantum stuff to orchestrate the observed results. This no causal influence assumption is called **locality**, and means that the test Bob chooses to make cannot influence Alice's stuff or vice versa. For Einstein there was no room for flexibility on this point. As I've already mentioned it was a fundamental principle in his own special theory of relativity. The consequences of causal influences travelling *faster* than light allow for a rather catastrophic breakdown in what could be called natural causality. Events in the future could influence those in the past! Einstein's relativity had banished the notion of instantaneous action-at-a-distance from physics and it was inconceivable that it would be reintroduced by quantum theory!

The EPR argument could thus be summarised as follows:

$$\begin{matrix}\text{Quantum}\\ \text{Mechanics}\end{matrix} + \text{Locality} \implies \begin{matrix}\text{Hidden}\\ \text{Variables}\end{matrix}$$

Note that this, of course, is an *argument* based on certain strong prejudices. One, as just mentioned, was locality and the other was realism, that we can think about reality in the atomic domain in the same terms as we can that of everyday experience.

Now recall the explanation we gave of the perfect correlations in terms of each entangled pair carrying with it one of four instruction sets, blue or red circles and blue or red squares. This is an explanation which satisfies the requirements both of locality and realism. In 1964 John Bell, a physicist at CERN, thought more deeply about this experimental arrangement. By slightly complicating it, effectively considering the possibility of testing for one of *three* properties, he proved a remarkable result. Quite generally, independent of quantum mechanics itself, he considered what correlations could arise in such a setup by just assuming what EPR held so dear, namely, locality and realism. He obtained a bound on the possible correlations, called **Bell's inequality**. Then, applying the quantum formalism to his setup he demonstrated that quantum mechanics predicted a *violation* of these bounds! Effectively what he demonstrated could be expressed as:

$$\begin{matrix}\text{Hidden}\\ \text{Variables}\end{matrix} + \text{Locality} \implies \begin{matrix}\textbf{Not}\ \text{Quantum}\\ \text{Mechanics}\end{matrix}$$

This opened up the possibility of carrying out an experimental test of whether Einstein was right. A test of whether nature was both local and realistic!

It was a happy coincidence that Bell's analysis appeared just as the technology required to realise such an experiment was emerging. Still, it wasn't until the french experimental physicist Alain Aspect's famous 1982 experiment that it was confirmed that, indeed, **nature obeys quantum mechanics** and therefore one of locality and realism must be given up.

Bell's result is not difficult to derive, it doesn't rely on any complicated mathematics. However it would require us to go beyond our simple colour/shape model of quantum stuff. Fortunately though, in 1989, Daniel Greenberger, Michael Horne and Anton Zeilinger discovered another way of demonstrating the fact that the combination of hidden variables and locality is not consistent with quantum mechanics and which it *is* possible to explain using our simple version of quantum theory. In any case this is a stronger and more dramatic result than the original Bell argument so don't feel short changed!

9. Quantum Mechanics Lesson 4 - The GHZ experiment

We now need three helpers, Alice, Bob and Charlie each alone in three far-flung galaxies to ensure there can be no question of one influencing the other. Each does exactly as Alice and Bob did for us before. Upon receiving a piece of quantum suff they randomly choose to test for colour or shape and record their result. As before, each observes an apparently random set of results. Having collected plenty of results they return to earth to compare what they found. It happens that they are unanimous in preferring (on purely aesthetic grounds) circles over squares and blue over red so blue and circle are deemed 'good'. Thanks to this preference they notice a remarkable pattern in their results.

Whenever two of them tested for shape and one for colour then they either had 0 or 2 'good' outcomes between them.

Now let's pause for a minute and think about what implications this might have in terms of our assumptions of locality and realism. Locality is of course dealt with by the fact that Alice, Bob and Charlie were all testing randomly on far flung galaxies. There was no chance (barring some sort of faster than light signalling) that they were in any way influencing each other. The realism assumption is more interesting. This amounts to the notion that the instructions of how to respond to whatever tests were performed were carried with the pieces of stuff to Alice, Bob and Charlie. Let's say the 'goodness value' of blue is 1 and 0 for red and that likewise the goodness value of circle is 1 and 0 for square. The three possible test combinations are exhibited in the following table.

Alice	Bob	Charlie
Colour	Shape	Shape
Shape	Colour	Shape
Shape	Shape	Colour

So, for example, it could be that Alice made a colour measurement whilst Bob and Charlie both carried out shape tests. In this case, then, the instructions being carried with the three pieces of quantum stuff arriving respectively at Alice, Bob and Charlie must be such that if Alice observed the outcome blue (good), whilst the result of Bob's shape test was circle (good) then we know that Charlie's shape test must have the outcome square (not good) ensuring a total of 2 good outcomes. Similarly, if Alice had instead found red (not good) whilst Bob had found square (not good) then again, to respect the observed correlations, the result of Charlie's shape test is determined to be square (not good) ensuring a total of 0 good outcomes. By the assumption of realism, each cell in the row has a goodness value, 1 or 0, and the observed correlations require the total for the row to be 0 or 2. But of course the type of experiment being carried out respectively by Alice, Bob and Charlie is in each case decided on the toss of a coin long after the pieces of quantum stuff have left earth and so it could also have been that Alice tested for shape, Bob for colour and Charlie for shape as in the second row or that both Alice and Bob chose to test for shape and Charlie for colour as in the third row. The assumption of realism, the notion that instruction sets are being carried with the individual pieces of quantum stuff, together with the fact that the respective measurement decisions are made randomly after the three pieces of entangled quantum stuff have left earth thus means that each cell in *each* row of the table has a goodness value of 0 or 1 and that summing these across the rows always gives the result 0 or 2. But this means that in the whole table the total goodness value of all the cells is even.

Now, notice that there are an even number, 6, of shape cells. This forces the sum of the goodness values of the three colour cells also to be even (an even number is either the sum of two evens or two odds, not a mixture). In other words we can already say with 100% certainty, based on the realist assumption, that *in any test in which Alice, Bob and Charlie each test for colour the total amount of good must be even.*

Feeling chuffed with themselves for this bit of Sherlock Holmes sleuthing they return to their data and scan for all the times they all tested for colour on the same triplet of pieces of quantum stuff. Dumbstruck they see that the data tells *precisely the opposite* story!

Whenever all of them tested for colour they either had 1 or 3 'good' outcomes between them.

Quantum mechanics predicts these two outcomes, utterly incompatible based on the twin assumptions of locality and realism, and experiments have confirmed the predictions of quantum mechanics.

It is a great irony that John Bell, who set in motion these tests of Einsteins dear principles of locality and realism, strongly believed that Einstein *should* have been right! Reflecting on the results of the original Aspect experiments he expressed his disappointment thus:

"*So for me, it is a pity that Einstein's idea doesn't work. The reasonable thing just doesn't work.*"

The quantum symbolics of the GHZ experiment are as follows. The three particles are prepared in a state, let's denote it $|GHZ\rangle$, which, expressed purely in terms of colour states looks like this,

$$|GHZ\rangle = \frac{1}{2}\left(|{\color{blue}\blacklozenge}\rangle_A |{\color{blue}\blacklozenge}\rangle_B |{\color{blue}\blacklozenge}\rangle_C + |{\color{blue}\blacklozenge}\rangle_A |{\color{red}\blacklozenge}\rangle_B |{\color{red}\blacklozenge}\rangle_C + |{\color{red}\blacklozenge}\rangle_A |{\color{red}\blacklozenge}\rangle_B |{\color{blue}\blacklozenge}\rangle_C + |{\color{red}\blacklozenge}\rangle_A |{\color{blue}\blacklozenge}\rangle_B |{\color{red}\blacklozenge}\rangle_C\right).$$

But this state can also be written using 1 colour state and 2 shape states as,

$$\begin{aligned}
|GHZ\rangle &= \frac{1}{2}\left(|{\color{blue}\blacklozenge}\rangle_A |\square\rangle_B |\bigcirc\rangle_C + |{\color{blue}\blacklozenge}\rangle_A |\bigcirc\rangle_B |\square\rangle_C + |{\color{red}\blacklozenge}\rangle_A |\bigcirc\rangle_B |\bigcirc\rangle_C + |{\color{red}\blacklozenge}\rangle_A |\square\rangle_B |\square\rangle_C\right) \\
&= \frac{1}{2}\left(|\bigcirc\rangle_A |{\color{blue}\blacklozenge}\rangle_B |\square\rangle_C + |\square\rangle_A |{\color{blue}\blacklozenge}\rangle_B |\bigcirc\rangle_C + |\bigcirc\rangle_A |{\color{red}\blacklozenge}\rangle_B |\bigcirc\rangle_C + |\square\rangle_A |{\color{red}\blacklozenge}\rangle_B |\square\rangle_C\right) \\
&= \frac{1}{2}\left(|\square\rangle_A |\bigcirc\rangle_B |{\color{blue}\blacklozenge}\rangle_C + |\bigcirc\rangle_A |\square\rangle_B |{\color{blue}\blacklozenge}\rangle_C + |\bigcirc\rangle_A |\bigcirc\rangle_B |{\color{red}\blacklozenge}\rangle_C + |\square\rangle_A |\square\rangle_B |{\color{red}\blacklozenge}\rangle_C\right)
\end{aligned}$$

I leave it as an exercise to confirm that the equality of each of these representations of the *same* state do indeed lead to the results Alice, Bob and Charlie recorded.

10. What Does It Mean?

In the discussion above I have barely entertained the possibility that some sort of 'spooky action at a distance', some kind of super-luminal signalling between the components of an entangled quantum system could be at work. This reflects my own personal view, and I must now come clean and admit that there is an active body of researchers, typically, but not exclusively, philosophers of science, for whom the universal validity of classical realism, the objective reality of properties of things, is sacrosanct. John Bell, who was most definitely a physicist, was in this camp and is still today, 26 years after his untimely death, the champion of their cause. They hold to the notion that the goal of science generally and physics in particular must be to provide a coherent account of *what is.* Not only is the classical notion of reality taken to be a self-evident fact of nature, without it, they believe physics must surely fall short of the standards set for itself by its own historic legacy. To quote Tim Maudlin, a prominent philosopher of science, "*I am only interested in physics insofar as it attempts to provide a clear and comprehensible account of* *what the physical world is.*" Thus, to these researchers, an entangled pair of pieces of quantum stuff are genuinely *physically* connected, regardless of how far apart they are, and carrying out a measurement on one piece indeed influences the other and this influence propagates instantaneously. The burning question is, of course, what are the implications for Einstein's relativistic description of space-time? To date this theory has never been found to be experimentally wanting and, being the inevitable consequence of a few very simple principles, feels solid. Moreover, it is surely significant that, apparently as a matter of principle, the 'strange communication' at work in entanglement cannot possibly be harnessed to usefully transfer information between two separated locations.[8] The followers of John Bell also point disparagingly at the prominent roles of observers and measurement in the quantum formalism. Are not observers governed by the laws of quantum mechanics too? Bell himself felt a need for a theory that "*neither needs nor is embarrassed by an observer.*"

Beyond those physicists focused on the philosophical foundations of quantum mechanics there is general acceptance that it is the assumption of realism which has to be given up. Besides the fact that the 'spooky action at a distance' is a magic out of our reach, they might also point to various other experimentally verified predictions of quantum mechanics, such as the violation of the so-called Leggett inequalities and the confirmation of the Kochen-Specker theorem all of which tend to support the idea that the classical notion of realism is not tenable at the atomic scale. I think many of them would agree with Feynman who in a 1983 BBC interview[9] suggested that clinging onto our intuitively satisfying notion of a reality 'out there', independent of us, is a "*deep prejudice...from being so used to large scale behaviour*". Even if they don't explicitly acknowledge it, they are adopting Bohr's position of physics being about *what can be said* rather than *what is.* Of course, giving up the universality of classical realism raises some serious and difficult questions. Perhaps most pressing among them is how it is that the realism we are familiar with emerges from the 'unreality' at the atomic scale?

Amongst those physicists who more or less align themselves with Bohr's philosophy and are concerned with the foundations of quantum theory, the remarkable success of the quantum formalism, with the observer and measurement so tightly integrated therein, hint at new ways of understanding our role in the construction of physical theories, of what, indeed, physical theories really *are*. Perhaps we should think of them rather as maps with which we may navigate the world around us rather then the world itself. This may seem like a retreat from the grand project of science as the unlocking of Nature's hidden mechanisms. I don't think so. I rather suspect that humans have gotten way ahead of themselves. Intoxicated by the undeniably impressive progress of science to date we foolishly believe that more of the same will continue to bear fruit and at the same remarkable rate. I suspect that the lesson of what we've learnt of the quantum world and more specifically of how we've been forced to interface with it is telling us that a radical change of perspective is required. We are not and cannot be separated from the world around us.

[8]This is simple to appreciate — take the case of a pair of gloves each one of which is placed blind into a box as discussed earlier. Upon opening his box in Melbourne Bob knows what's in Alice's box instantly. But, without calling Alice by some conventional (slower than light speed!) means this information cannot usefully be used. For Alice, without receiving such a call from Bob, there is still a 50:50 chance of finding a right or left hand glove in her box. To her, nothing has changed.

[9]"Physics is fun to imagine" can be viewed at `https://www.ted.com/talks/richard_feynman`

At the end of the interview with Feynman I quoted from earlier he said something which resonates deeply with my own personal 'faith' and which I feel is an appropriate way to end our story.

"*I think Nature's imagination is so much greater than man's she's never going to let us relax.*"