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Specialist Maths Units 3/4

Statistics

Practice Questions

- Linear combinations of random variables
- Distribution of sample means
- Hypothesis testing for the mean

Short Answer Questions

Question 1

A company operates a telephone helpline for its clients. The waiting time, in minutes, for clients using the helpline is normally distributed with mean μ.
A random sample is taken of 100 of the company's client who use the helpline and an appropriate 95% confidence interval for μ is $48.04 < \mu < 51.96$

a) Find the mean $\overline{x}$ and the standard deviation s for this sample.

b) The company upgrades its telephone helpline by outsourcing it to a call centre business. The waiting times, in minutes, for callers to this call centre are normally distributed with an advertised mean waiting time of 5.2 minutes and a standard deviation of 1.5 minutes. After the upgrade, the company is suspicious that waiting times on average are longer than those suggested by the advertising. It randomly samples 25 of its clients and finds that the mean waiting time is 5.8 minutes. Assume that the population standard deviation remains at 1.5 minutes.

i. Write down an appropriate null and alternative hypothesis to test whether the mean waiting time is longer than that advertised by the call centre business.

ii. Find the p value of this test.

iii. Hence explain whether or not the null hypothesis should be rejected or not rejected at the 5% level of significance.

Question 2

The mass of the oranges grown in an orchid is normally distributed with a mean of mug and a standard deviation of 13g.
A sample of 169 oranges has a mean mass of 150g and the standard deviation remains at 13g.

a) Find an approximate 90% confidence interval for the population mean, given that $\Pr(z > 1.645) \approx 0.05$ where z has the standard normal distribution. Give your answer to one decimal place.

b) Another orchard produces oranges with mean mass of 160g and standard deviation of 6g. The owner of the orchard introduces a new fertiliser, which is intended to increase the mass of the oranges. After using new fertiliser, a sample of 144 oranges has a mean mass of 161g and the standard deviation remains 6g.

i. Write down the appropriate null and appropriate hypotheses to test whether the mean mass of the oranges has increased as a result of the new fertiliser.

ii. Find an appropriate p value of this test, correct to three decimal places.

iii. Explain why the null hypothesis should be rejected at the 5% level of significance.

Question 3

The study scores of a large population of VCE Specialist Mathematics students have a mean of 30 and a standard deviation of 7. A random sample of 100 students is to be selected from the population. The sample mean is denoted by $\overline{X}$.

a) State $E(\overline{X})$.

b) Find $\sigma(\overline{X})$.

The sample size is now increased by a factor of 2.

c) Determine how $E(\overline{X})$ and $\sigma(\overline{X})$ would change as a result of this increase in sample size.

Question 4

A farmer grows peaches, which are sold at a local market. The mass, in grams, of peaches produced on this farm is known to be normally distributed with a variance of 25. A bag of 36 peaches is found to have a total mass of 4320g.
Based on this sample of 36 peaches, calculate an approximate 99.7% confidence interval for the mean mass of all peached produced on this farm. Use an integer multiple of the standard deviation in your calculations.

Question 5

The volume of soft drink dispensed by a machine into bottles varies normally with a mean of 300 mL and a standard deviation of 15 mL. The soft drink is sold in packs of nine bottles.

Find the approximate probability that the mean volume of soft drink per bottle in a randomly selected nine-bottle pack is less than 285 mL. Give your answer as a percentage to 3 decimal places.

Multiple Choice Questions

Question 1

A popular brand of pasta sells its penne in packets labelled as 750g. A sample of 40 packets of penne have a mean weight of 748.6 grams and a standard deviation of 2.7g. An approximate 95% confidence interval of a population mean μ is closest to

A. $747.8 < \mu < 749.4$

B. $748.2 < \mu < 749.0$

C. $748.5 < \mu < 751.5$

D. $747.9 < \mu < 748.1$

E. $749.1 < \mu < 751.0$

Question 2

A census found that mean weekly earnings for adult full time workers in a particular region was \$1554 with a standard deviation of \$162. A random sample of 50 adult full time workers in this region was taken. The probability that the mean weekly earnings for this group was less than \$1500 is closest to

A. 0.0092

B. 0.0522

C. 0.2389

D. 0.3694

E. 0.4986

Question 3

A statistical test was conducted involving a null hypothesis H_0 and an alternative hypothesis H_1. The results of the test indicated that H_0 should be rejected. Which one of the following statements could not be correct?

A. A correct decision has been made.

B. A type 1 error has been made.

C. An incorrect decision has been made.

D. A type 2 error has been made.

E. H_1 is true.

Question 4

Given that X is a normal random variable with mean 10 and standard deviation 8, and that Y is a normal random variable with mean 3 and standard deviation 2, the random variable Z defined by $Z = X - 3Y$ will have mean μ and standard deviation σ given by

A. $\mu = 1, \sigma = 28$

B. $\mu = 19, \sigma = 2$

C. $\mu = 1, \sigma = 2\sqrt{7}$

D. $\mu = 19, \sigma = 14$

E. $\mu = 1, \sigma = 10$

Question 5

The mean study score for a large VCE study is 30 with a standard deviation of 7. A class of 20 students may be considered as a random sample drawn from this cohort. The probability that the class mean for the group of 20 exceeds 32 is

A. 0.1007

B. 0.3875

C. 0.3993

D. 0.6125

E. 0.8993

Question 6

A type I error would occur in a statistical test were

A. H_0 is accepted when H_0 is false.

B. H_1 is accepted when H_1 is true.

C. H_0 is rejected when H_0 is true.

D. H_1 is rejected when H_1 is true.

E. H_0 is accepted when H_0 is true.

Extended Response Questions

Question 1

If the mean weight of each passenger's luggage on flights between Melbourne and Sydney is 19kg with standard deviation 6kg. What is the mean and standard deviation for the total weight of luggage of 70 passengers? What is the 3σ confidence interval for this weight? Assuming the total weight is approximately normally distributed, what is the probability that the total weight will exceed 1400kg?

Question 2

In an experiment to test the effectiveness of a new drug to lower blood pressure, suppose that the blood pressure of 20 people taking the drug averaged at 128 on a certain scale and measurement. Additionally, independently, 30 people not taking the drug averaged at 130. Assuming the standard deviation for an individual person is 2.5 in either case, what is the standard deviation of the differences in means?

Question 3

The reaction times of 200 randomly selected drivers were found to have a sample mean of 0.83 seconds, and a sample standard deviation of 0.20 seconds. Find a 95% confidence interval for the mean reaction time of the whole population of drivers.

Question 4

In a random sample of 100 tyres produced by a certain company, 20 did not meet the company's quality control standards. Construct a 95% confidence interval for the overall population which do not meet the standards, given the variance for the proportion of the tyres which do not meet the standards is 0.0256. What sample size should be taken in order to obtain a 95% confidence interval of width 0.1?

Question 5

The weight of J of an empty jar form a factory has a mean of 252.3g and a standard deviation of 3.6g, and there are Rg of raspberry jam in each jar, where R has a mean of 362.5g and a standard deviation of 3.6g. It is assumed that R and J are independent and both are normally distributed.

a) Let W be the full weight of each jar ($W = R + J$)

i. Find the expected value of W, $E(W)$.

ii. Find the variance of W, $\text{Var}(W)$, correct to one decimal place.

iii. Hence, find the standard deviation of W, $\text{SD}(W)$, correct to one decimal place.

b) The factory also produced jars of Vegemite. The weight V is also normally distributed. A full jar of Vegemite has a mean weight of 612g and a standard deviation of 6.4g. There are twelve jars of Vegemite packed into a container and all jars of Vegemite are independent from each other.

i. Find the expected value of the weight of a container of Vegemite (Assume that the container itself has no weight).

ii. Find the standard deviation of the weight of a container of Vegemite.

iii. The container will burst if the total weight inside them is more than 7.4kg. Find the probability, correct to four decimal places, that a container of 12 jars of Vegemite will break.

iv. State the 95% confidence interval for the estimation of the mean weight of a jar of Vegemite in a container, giving your answer correct to 1 decimal place.

Question 6

For Australian children it is known that the reaction times of boys and girls each follow a normal distribution where

- Boys had an average reaction time of 0.66 seconds, with a standard deviation of 0.44 seconds.
- Girls had an average reaction time of 0.80 seconds, with a standard deviation of 0.32 seconds.

a) Four boys are randomly chosen and their reaction times are measured. Find the probability that the sum of their reaction times is less than 2.5 seconds. Give your answer correct to two decimal places.

b) Let D be the random variable, 'Difference between the reaction times of an Australian boy and girl.'

i. Find, correct to two decimal places, the mean and variance of D.

ii. Find the probability that a randomly chose girl has a faster reaction time than a randomly chose boy. Give your answer correct to two decimal places.

c) A researcher wonders whether there has been any change in the reaction times of Australian girls after a Youtube clip of a girl catching a ruler at lightning speed went viral. The researcher collected a random sample of n girls, tested their reaction times and then constructed an approximate 90% confidence interval of (0.54, 0.79) for their reaction times. The population standard deviation was assumed to stay to same.
Find the value of n.

Question 7

A tyre manufacturer produces thirteen inch car tyres using rubber and other components, The random variable R represents the weight, in kg, of rubber in one of these tyres. The random variable C represents the weight, in kg, of other components in one of these tyres.

Both variables are normally distributed. $R \sim N(2.2, 0.18)$ and $C \sim N(4.9, 0.31)$. Let W represent the total weight of four of these thirteen inch car tyres.

a) Find the mean and standard deviation of W.

The tyre manufacturer also produces fifteen inch car tyres. The distribution of the weight of these tyres is normal and it is claimed by the manufacturer that $\mu = 8.6$kg. The motoring organisation tests these claims by taking a random sample of 36 tyres. It finds the mean of the sample to be $\overline{x} = 8.4$kg and the standard deviation to be $s = 0.54$kg. Using this sample data it checks the manufacturer's claims. It assumes that the population standard deviation of these tyres (σ) can be estimated with sufficient accuracy by s.

b) Write down two hypotheses, H_0 and H_1, that could be used to test whether the mean weight of the tyres is less than that being claimed by the manufacturer.

c) Find the p value of this statistical test. Give your answer correct to three decimal places.

d) State whether H_0 should be rejected or not at the 5% level of significance. Give a reason to support your answer.

The motoring organisation decides to repeat this test of the manufacturer's claim that $\mu = 8.6$kg, but this time it tests at a 1% level of significance. Use the same hypotheses, it again takes a random sample of 36 tyres. The standard deviation of this seconds sample is found again to be 0.54kg. The motoring organisation concludes that according to this seconds sample, the null hypothesis should be rejected at the 1% level of significance.

e) Find the minimum value of $\overline{x}$ that the motoring organisation could have found in its second sample. Give your answer correct to three decimal places.

Question 8

The BigSaw Company uses cut down trees to make decking timber. The decking timber is sold as having a length of 3.6 metres although the actual length varies slightly. The distribution, D, of the length is known to be normal with a standard deviation 2.1cm.

a) The company wants to check that their cutting machine was working accurately and the the mean was 3.6 metres, so they took a random sample of 5 pieces of the decking timber and measured the lengths in cm, with the following results

362.1 355.2 361.4 359.3 357.5

i. Write down H_0 and H_1, suitable hypotheses to test the length of decking timber.

ii. Find the p values for this test, correct to three decimal places.

iii. State with a reason whether H_0 should be rejected or not at a 5% significance level.

iv. State the type of error if the conclusion in **part a.iii.** was incorrect.

b) The manager is still concerned so they do another test, this time taking a sample of 50 pieces of decking timber. The mean for this sample was 359.3cm and the standard deviation remained the same at 2.1cm.

i. Find a such that the p value of this test is given by the expression $\Pr(|Z| > a)$, where Z is the normal standard distribution.

ii. Hence, find the p value, correct to 3 decimal places, to test the null hypothesis using the new sample mean at a 5% significance level.

iii. Hence, state with reason, whether it is sufficient to reject the null hypothesis H_0 at the 5% significance level.

iv. State the type of error if the conclusion in **part b.iii.** was incorrect

c) The company also makes wood posts of unknown mean length. A sample of 50 posts has a mean value of 1.2m with a standard deviation of 0.07m.

i. Find the 99% confidence interval for the estimation of the mean post length. Give your answer correct to 3 decimal places.

ii. Assuming the standard deviation of 0.07m, what is the smallest sample size that is needed to be 95$ confident that the sample mean is correct with a margin of error of ± 0.01m from the population mean.