THE INSTITUTE FOR ENQUIRING MINDS
redefining access
resetting attitudes
enquiringminds.org
info@enquiringminds.org
+61 448 218652

Specialist Maths Units 3/4

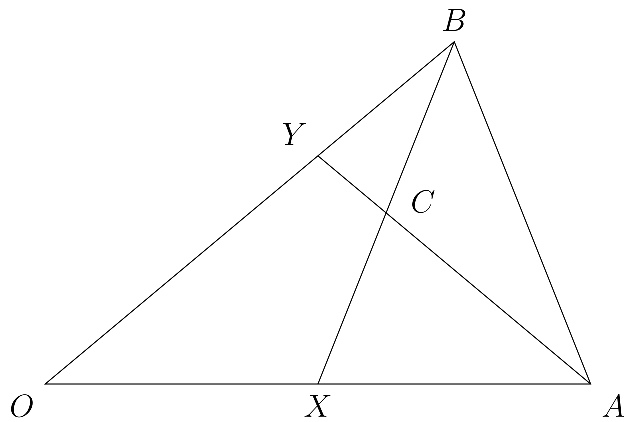
Vectors

Practice Questions

Short Answer Questions

Question 1

For the triangle OAB shown, let the sides $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$ and X be the midpoint of $\overrightarrow{OA}$ and Y be the midpoint of $\overrightarrow{OB}$.



a) Express the medians ($\overrightarrow{XB}$ and $\overrightarrow{YA}$) in terms of vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

b) Given that $\overrightarrow{XC} = \alpha\overrightarrow{XB}$ and $\overrightarrow{YC} = \beta\overrightarrow{YA}$, write an expression of $\overrightarrow{XC}$ in terms of α, a and b.

c) Hence prove that α and β are equal to $\dfrac{1}{3}$

Question 2

The triangle ABC has vertices at $A(5,3,3), B(3,5,2)$ and $C(-1,2,4)$.

a) Find the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

b) Hence, show that $\triangle ABC$ is a right-angled triangle.

c) Calculate the area $\triangle ABC$.

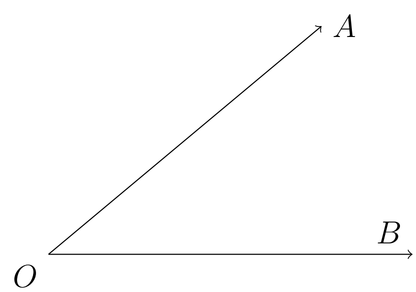
d) Find the point D that makes the parallelogram $ABCD$.

Question 3

If $|\underset{\sim}{a}| = 9$ and $|\underset{\sim}{b}| = 7$ and $\underset{\sim}{a} \cdot \underset{\sim}{b} = 12$, find $|\underset{\sim}{a} + \underset{\sim}{b}|$.

Question 4

Given $\overrightarrow{OA} = 4\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\overrightarrow{OB} = 5\underset{\sim}{i} + 3\underset{\sim}{k}$.



a) Find the unit vectors $\hat{\overrightarrow{OA}}$ and $\hat{\overrightarrow{OB}}$.

b) Hence, find a unit vector which bisects the angle AOB.

Question 5

Given that $\utilde{u} = x_1\utilde{i} + y_1\utilde{j} + z_1\utilde{k}$ and $\utilde{v} = x_2\utilde{i} + y_2\utilde{j} + z_2\utilde{k}$, show that $u + v$ and $u - v$ are perpendicular if $|\utilde{u}| = |\utilde{v}|$.

Question 6

Given that $\utilde{a} = 6\utilde{i} - 2\utilde{j} + \utilde{k}$ and $\utilde{b} = 3\utilde{i} + \utilde{j} + 2\utilde{k}$, find two vectors $\utilde{c}$ and $\utilde{d}$ such that *all three* of the following conditions apply:

- $\utilde{a} = \utilde{c} + \utilde{d}$
- $\utilde{d}$ is parallel to $\utilde{b}$
- $\utilde{c}$ is perpendicular to $\utilde{b}$

Question 7

A line in a 3D space has the direction vector $2\utilde{i} + 4\utilde{j} - 3\utilde{k}$ and contains the point $P(1, -2, 0)$. The point $Q(2, -1, 4)$ is not on the line itself. Find the shortest distance between the line and the point Q.

Question 8

Resolve the vector $3\underset{\sim}{i} + 3\underset{\sim}{j} + 3\underset{\sim}{k}$ into two vector components, one which is parallel to the vector $-4\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}$ and one which is perpendicular to it.

Question 9

Given $\underset{\sim}{u} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{v} = -\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{w} = a\underset{\sim}{i} + b\underset{\sim}{j} - 6\underset{\sim}{k}$

a) Find the value of a and b if $\underset{\sim}{w}$ is perpendicular to both $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

b) Resolve $\underset{\sim}{v}$ into components parallel and perpendicular to $\underset{\sim}{u}$.

Question 10

If A and B have the Cartesian coordinates $(1, 0, -1)$ and $(2, -1, 1)$ respectively.

a) If θ is the angle between the vectors $\overrightarrow{OB}$ and $\overrightarrow{AB}$, show that $\cos(\theta) = \frac{5}{6}$.

b) Find the exact area of $\triangle OAB$.

Question 11

Find the coordinates of the point P, which lies on the line AB and divides it in the ratio, 2:5, where $A = (-4, 2, 5)$ and $B = (10, 9, -2)$.

Question 12

If $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - 6\underset{\sim}{k}$, find the cosine of the angle between these two vectors.

Question 13

Find the shortest distance between the points $P(3,4,7)$ and the line joining $A(1,1,1)$ and $B(2,3,3)$.

Question 14

Find a vector of length 3 which is perpendicular to $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 4\underset{\sim}{k}$.

Question 15

Consider the vector $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}, \underset{\sim}{b} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{c} = 4\underset{\sim}{i} - \underset{\sim}{j} + 5\underset{\sim}{k}$.

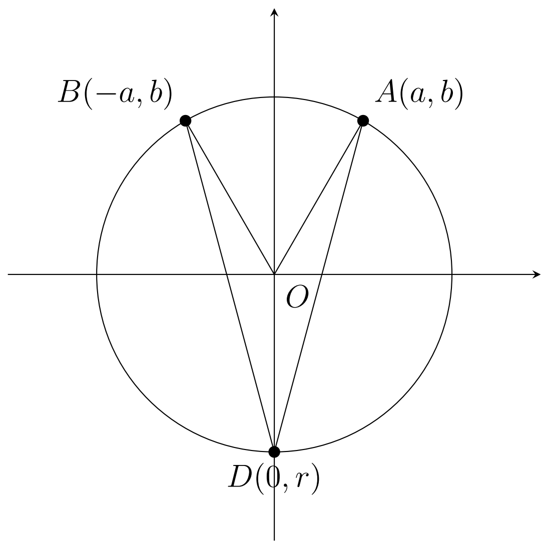
a) Find the unit vector in the direction of $\underset{\sim}{a}$.

b) Verify that $\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{c}$.

Question 16

The diagram below. Shows a circle of radius r. With centre at the origin O, the three points A, B and D all lie on the circle and have coordinates $A(a,b), B(-a,b)$ and $D(0,-r)$ where a, b and r are all positive real constants.

a) Let θ be the angle between vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Using vectors express $\cos(\theta)$ in terms of a and b.



b) Let α be the angle between $\overrightarrow{DA}$ and $\overrightarrow{DB}$. Using vectors show that $\cos(\alpha) = \dfrac{b}{\sqrt{a^2+b^2}}$

c) Hence show that $2\alpha = \theta$.

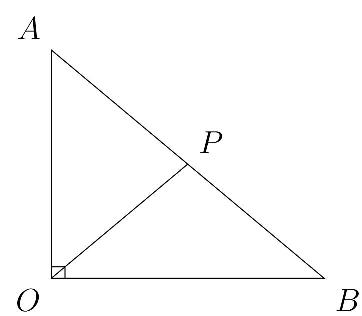
i. A unit vector perpendicular to both $\underset{\sim}{b}$ and $\underset{\sim}{c}$ is given by $\underset{\sim}{n} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, where x, y and $z \in \mathbb{R}$ and $x < 0$. Find $\underset{\sim}{n}$.

ii. The points A, B and C have position vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively to an origin O, A, B and C are four of the eight vertices of a cuboid. Find the volume of a cuboid.

Question 17

The points A, B and P have position vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{p}$ respectively relative to an origin O and $\angle AOB$ is a right angle. It is known that $\overrightarrow{AP} = k\overrightarrow{AB}$, where k is a real constant. The relative positions of A, B and P are shown in the diagram below.

a) If $\angle AOP = \theta$ show that $\cos\theta = \dfrac{(1-k)|\underset{\sim}{a}|}{|P|}$

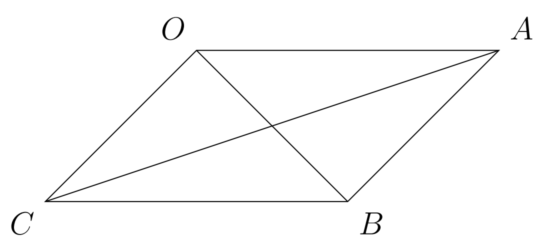


b) If $\angle BOP = \alpha$ find a similar expression for $\cos(\alpha)$.

c) If $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\theta = \alpha$ find the value of k.

Question 18

In the parallelogram shown below $\overrightarrow{OA} = \underset{\sim}{a}, \overrightarrow{OC} = \underset{\sim}{c}$ and $\overrightarrow{OM} = \underset{\sim}{m}$, where M is the point of intersection of the two diagonals.



Let $\overrightarrow{OM} = p\overrightarrow{OB}$ and $\overrightarrow{CM} = q\overrightarrow{CA}$, where $p, q \in \mathbb{R}$.

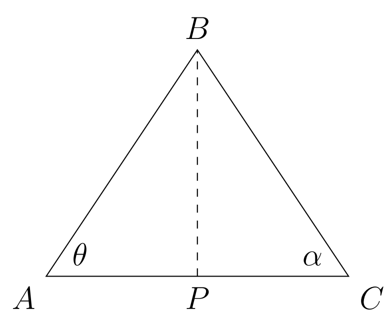
a) **i.** Show that $\underset{\sim}{m} = p(\underset{\sim}{a} + \underset{\sim}{c})$

ii. Show that $\underset{\sim}{m} = q\underset{\sim}{a} + (1-q)\underset{\sim}{c}$.

b) Hence, prove that the diagonals of a parallelogram bisect each other at point M.

Question 19

The diagram below shows an isosceles triangle with equal sides AB and BC.
Let $\overrightarrow{AB} = \underset{\sim}{b}$, $\overrightarrow{AC} = \underset{\sim}{a}$ and $\overrightarrow{AP} = m\overrightarrow{AC}$, where m is a real constant.
The angle $\angle APB$ is a right angle.



a) By considering the vectors $\overrightarrow{PA}$ and $\overrightarrow{PB}$ show that $a \cdot b = m|a|^2$.

b) Show that $\cos(\alpha) = \dfrac{a \cdot a - a \cdot b}{|a||b|}$.

c) Find a similar expression for $\cos(\theta)$ involving the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

d) Hence, show that $AP = PC$.

Question 20

Consider a triangle ABC and let $\underset{\sim}{b} = \overrightarrow{AB}$ and $\underset{\sim}{c} = \overrightarrow{AC}$.
Let M and N be the midpoints of sides BC and AC respectively.

a) **i.** Show that $\overrightarrow{AM} = \dfrac{1}{2}(\underset{\sim}{b} + \underset{\sim}{c})$

ii. Find an expression for $\overrightarrow{BN}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

Let G be the point at which the medians $\overrightarrow{AM}$ and $\overrightarrow{BN}$ intersect.
Let $\overrightarrow{AG} = \lambda\overrightarrow{AM}$ and $\overrightarrow{BG} = \mu\overrightarrow{BN}$.

b) Prove that $\lambda = \mu = \dfrac{2}{3}$.

c) Hence prove that the medians of a triangle are concurrent

Let the point O be the origin

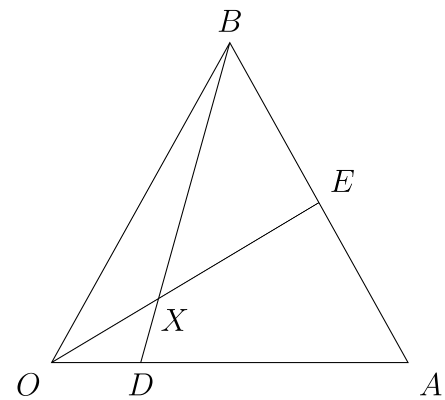
d) Prove that $\overrightarrow{OG} = \dfrac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$

Question 21

In the figure below, points A and B have position vectors α and β respectively, relative to an origin O. The point D is such that $\overrightarrow{OD} = \lambda\overrightarrow{OA}$ and the point E is such that $\overrightarrow{AE} = \mu\overrightarrow{AB}$. The line BD and OE intersect at X:

It is also known that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OE}$ and $\overrightarrow{XB} = \frac{4}{5}\overrightarrow{DB}$.

a) Express $\overrightarrow{XB}$ in terms of a, b and λ.



b) Express $\overrightarrow{OX}$ in terms of a, b and μ.

c) Using the fact that a and b are linearly independent, find the values of λ and μ.

Multiple Choice Questions

Question 1

In the Cartesian plane, a vector perpendicular to the line $3x + 2y + 1 = 0$ is:

A. $3\underset{\sim}{i} + 2\underset{\sim}{j}$

B. $-\frac{1}{2}\underset{\sim}{i} + \frac{1}{3}\underset{\sim}{j}$

C. $2\underset{\sim}{i} - 3\underset{\sim}{j}$

D. $\frac{1}{2}\underset{\sim}{i} - \frac{1}{3}\underset{\sim}{j}$

E. $2\underset{\sim}{i} + 3\underset{\sim}{j}$

Question 2

Let $\underset{\sim}{m} = 4\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{n} = \underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$. A unit vector in the direction of $m - 2n$ is:

A. $\frac{1}{7}(2\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k})$

B. $\frac{1}{3}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$

C. $\frac{1}{\sqrt{17}}(2\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k})$

D. $\frac{1}{\sqrt{41}}(6\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k})$

E. $\frac{1}{\sqrt{29}}(3\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k})$

Question 3

Which of the following is not an expression for the vector resolute of vector $\underset{\sim}{a}$ in direction of $\underset{\sim}{b}$.

A. $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}}\, \underset{\sim}{b}$

B. $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|^2}\, \underset{\sim}{b}$

C. $\left(\underset{\sim}{a} \cdot \dfrac{\underset{\sim}{b}}{|\underset{\sim}{b}|}\right)\left(\dfrac{\underset{\sim}{b}}{|\underset{\sim}{b}|}\right)$

D. $(\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}})\, \hat{\underset{\sim}{b}}$

E. $\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}}$

Question 4

Consider the three vectors below:

$$\underset{\sim}{m} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$$
$$\underset{\sim}{n} = 8\underset{\sim}{i} + a^2\underset{\sim}{j} + a^3\underset{\sim}{k}$$
$$\underset{\sim}{p} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$$

A. The vectors are linearly independent for all values of a.

B. The vectors are linearly dependent for all values of a.

C. The vectors are linearly independent exactly when $a = 2$.

D. The vectors are linearly independent exactly when $a \neq 2$.

E. None of the above.

Question 5

The position vectors of three points, A, B and C relative to the origin are given by a, b and c respectively.
If $3a - 5b + 2c = 0$ and $\overrightarrow{AB} = m\overrightarrow{BC}$, where m is a real constant, then.

A. $m = \dfrac{3}{2}$ and a, b, c are linearly dependent

B. $m = \dfrac{2}{3}$ and a, b, c are linearly independent

C. $m = -\dfrac{2}{3}$ and a, b, c are linearly independent

D. $m = \dfrac{2}{3}$ and a, b, c are linearly dependent

E. $m = \dfrac{3}{2}$ and a, b, c are linearly independent

Question 6

Let $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{j} + 2\underset{\sim}{k}$. A vector of magnitude $|\underset{\sim}{a}|$ in the direction of $\underset{\sim}{a} - \underset{\sim}{b}$, is:

A. $\dfrac{1}{3}(2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k})$

B. $\dfrac{-\sqrt{6}}{3}(-2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k})$

C. $\dfrac{\sqrt{6}}{6}(2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k})$

D. $\sqrt{6}(2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k})$

E. $\dfrac{\sqrt{6}}{3}(-2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k})$

Question 7

Let $\underset{\sim}{u} = 2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{v} = a\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$. If the scalar resolute of v in the direction of u is 1, that value of a is:

A. $-\frac{3}{2}$

B. $-\frac{2}{3}$

C. 3

D. $\frac{2}{3}$

E. $\frac{3}{2}$

Question 8

The vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$ are all non-zero vectors. If $\underset{\sim}{a}$ is perpendicular to$\underset{\sim}{c}$, which one of the following statements must be true.

A. $a \cdot a = c \cdot c$

B. $a \cdot b = b \cdot c$

C. $(a \cdot c)\ b = b$

D. $a \cdot (a + c) = |a|^2$

E. $(a + c) \cdot (a - c) = |a| - |c|$

Question 9

The vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are linearly independent. Points A and B are represented by $2\underset{\sim}{u} + x\underset{\sim}{v}$ and $y\underset{\sim}{u} + 5\underset{\sim}{v}$ respectively. If $\overrightarrow{AB} = (x+1)\underset{\sim}{u} + (y+1)\underset{\sim}{v}$ then:

A. $x = 1, y = 0$

B. $x = 4, y = 7$

C. $x = \frac{3}{2}, y = \frac{5}{2}$

D. $x = 0, y = 3$

E. $x = \frac{1}{2}, y = \frac{7}{2}$

Question 10

Points P, Q, R and M are such that $\overrightarrow{PQ} = 5\underset{\sim}{i}$, $\overrightarrow{PR} = \underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\overrightarrow{RM}$ is parallel to $\overrightarrow{PQ}$, so that $\overrightarrow{RM} = \lambda\underset{\sim}{i}$ where λ is a constant. The value of λ for which $\angle RQM$ is a right angle is:

A. 0

B. $\frac{19}{4}$

C. $\frac{21}{4}$

D. 10

E. 6

Question 11

Let $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{a}$ be any vector such that $\underset{\sim}{a} \cdot \underset{\sim}{b} = 2$. The smallest possible value of $|\underset{\sim}{a}|$ is:

A. 2

B. -1

C. $2\sqrt{6}$

D. $\frac{\sqrt{6}}{3}$

E. $-\frac{\sqrt{6}}{3}$

Extended Response Questions

Question 1

Point A has position vector $\underset{\sim}{a} = m\underset{\sim}{i} + 2\underset{\sim}{j}$, point B has position vector $\underset{\sim}{b} = \underset{\sim}{i} + 6\underset{\sim}{j}$, point C has position vector $\underset{\sim}{c} = n\underset{\sim}{i} + 6\underset{\sim}{j}$ and point D has position vector $\underset{\sim}{d} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ relative to the origin O, where m and n are real numbers.

a) Find $\overrightarrow{AB}$ and $\overrightarrow{AD}$ in terms of m.

b) Hence, use a vector method to show that the values of m and n are -2 and 6 respectively such that $ABCD$ is a rhombus.

c) Show that $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{BD}$.

d) Find $\overrightarrow{AE}$, the vector resolute of $\overrightarrow{AB}$ parallel to $\overrightarrow{AD}$.

e) Let $\theta°$ be the angle between vector $\overrightarrow{AB}$ and vector $\overrightarrow{AD}$ where $\theta < 90°$

Question 2

a) Find the following vectors in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

i. $\overrightarrow{AB}$

ii. $\overrightarrow{BP}$ where $\overrightarrow{OP} = \dfrac{1}{3}\overrightarrow{OA}$.

iii. $\overrightarrow{BQ}$ where $\overrightarrow{PQ} = \overrightarrow{BP}$.

b) Now consider the particular triangle OAB with $\overrightarrow{OA} = 6\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\overrightarrow{OB} = 4\underset{\sim}{i}$, with O being the origin.
If as above, $\overrightarrow{OP} = \dfrac{1}{3}\overrightarrow{OA}$ and $\overrightarrow{PQ} = \overrightarrow{BP}$. Show that $\overrightarrow{OQ} = 2\underset{\sim}{j}$

c) **i.** Find $\overrightarrow{OR}$ such that $OQRB$ forms a rectangle.

ii. Hence show that $\overrightarrow{OR} = \mu\overrightarrow{OA}$ and find the value of μ.

d) C is a point such that $|\overrightarrow{OC}| = \mu|\overrightarrow{AC}|$ and $\overrightarrow{OC} = 3\underset{\sim}{i} + p\underset{\sim}{j} + q\underset{\sim}{k}$ and $\angle AOC = 60°$, Find the values of p and q.

Question 3

AOC is a triangle, where F and E is the midpoint of $\overrightarrow{OA}$ and $\overrightarrow{AE}$ respectively. The medians CF and OE intersect at X.
Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

a) Find $\overrightarrow{CF}$ and $\overrightarrow{OE}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

b) **i.** If $\overrightarrow{OE}$ is perpendicular to $\overrightarrow{AC}$, prove that triangle OAC is isosceles.

ii. If furthermore $\overrightarrow{CF}$ is perpendicular to $\overrightarrow{OA}$, find the magnitude of angle AOC, and hence prove that triangle is AOC is equilateral.

c) H and K are the midpoints of OE and CF respectively.

i. Show that $\overrightarrow{HK} = \lambda \underset{\sim}{c}$ for $\lambda \in \mathbb{R}\backslash\{0\}$ and $\overrightarrow{FE} = \mu \underset{\sim}{c}$ for $\mu \in \mathbb{R}\backslash\{0\}$

ii. Give reasons why triangle HXK is similar to triangle EXF.

iii. Hence prove that $OX : XE = 2 : 1$.

Question 4

The diagram below shows that trapezium $OAEC$, in which CE is parallel to and four times as long as OA. B is the midpoint of CE. Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

a) Express in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$

i. $\overrightarrow{CE}$

ii. $\overrightarrow{CB}$

iii. $\overrightarrow{AB}$

iv. $\overrightarrow{OE}$

Let D be the point on $\overrightarrow{AB}$ such that $\overrightarrow{AD} : \overrightarrow{DB} = 1 : 2$

b) Express in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$

i. $\overrightarrow{AD}$

ii. $\overrightarrow{OD}$

c) Use your results from parts **a.** and **b.** to explain why O, D, E are collinear.

Question 5

The vertices of a tetrahedron $OABC$ are given by the position vectors,
$\overrightarrow{OA} = 4\underset{\sim}{i} + 2\underset{\sim}{j}, \overrightarrow{OB} = 3\underset{\sim}{j}$ and $\overrightarrow{OC} = \sqrt{\lambda}\underset{\sim}{k}$.

a) Find the magnitude of $\angle AOB$ in degrees and correct to 2 decimal places.

b) Find:

i. $\overrightarrow{AC}$

ii. $\overrightarrow{BC}$

c) Given that $\angle ACB = 30°$, find the value/s of λ.

d) For a different tetrahedron where $\sqrt{\lambda} = 6$ giving $\overrightarrow{OA} = 4\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\overrightarrow{OC} = 6\underset{\sim}{k}$.
Let X be a point on the line segment AB
Find the coordinates of X such that CX is perpendicular to AB.

Question 6

Points $O(0,0,0)$, $A(6,2,-1)$ and $B(4,-3,3)$ form the vertices of a triangle as shown below (the z-axis comes out of the page).
The position vectors $\overrightarrow{OA} = \underset{\sim}{a} = 6\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\overrightarrow{OB} = \underset{\sim}{b} = 4\underset{\sim}{i} - 3\underset{\sim}{j} + 3\underset{\sim}{k}$ are indicated. AP is an altitude of triangle OAB.

a) Find the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$.

b) Hence find the length of the altitude AP.

c) Find the exact area of triangle OAB.

d) Find a vector that bisects the angle AOB.

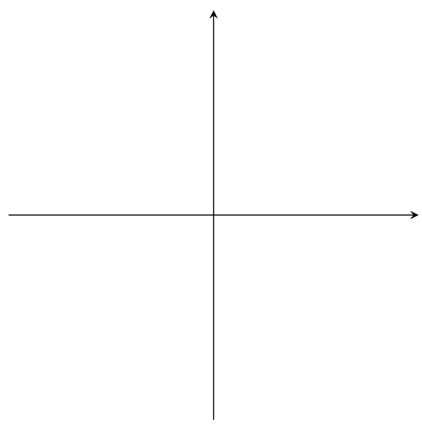
e) $OECD$ is a parallelogram. Given that $\overrightarrow{CD}$ is perpendicular to $\overrightarrow{OE}$, prove using vectors that $OCED$ is a rhombus.

Question 7

a) If $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} - 5\underset{\sim}{j}$, and given $|\underset{\sim}{r} - \underset{\sim}{a}| = |\underset{\sim}{r} - \underset{\sim}{b}|$

i. Find the Cartesian equation and describe the path of the set of points with position vector $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$.

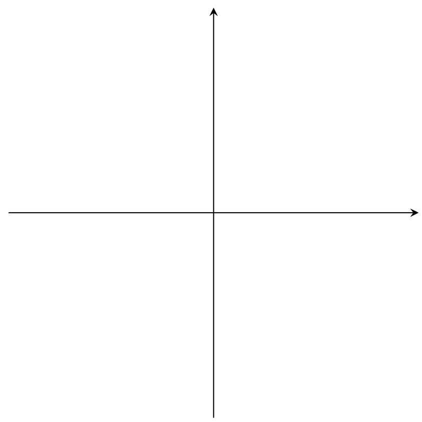
ii. Sketch a graph of this path, clearly showing its relationship with the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.



b) If $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} - 5\underset{\sim}{j}$, and given $(\underset{\sim}{r} - \underset{\sim}{a}) \cdot (\underset{\sim}{r} - \underset{\sim}{b}) = 0$

i. Find the Cartesian equation and describe the path of the set of points with position vector $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$.

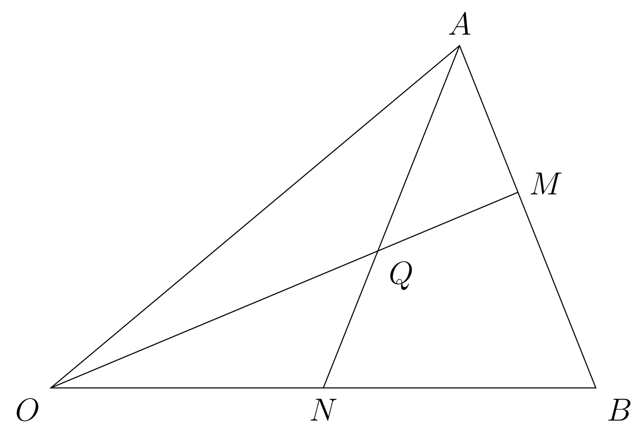
ii. Sketch a graph of this path, clearly showing its relationship with the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.



Question 8

Consider triangle OAB where O is the origin, $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

P, M and N are midpoints of OA, AB and OB respectively. The point Q is the intersection point of the intervals OM and AN.
Given $\overrightarrow{OQ} = \lambda\overrightarrow{OM}$ and $\overrightarrow{AQ} = \mu\overrightarrow{AN}$.



a) **i.** Find $\overrightarrow{OQ}$ in terms of $\lambda, \underset{\sim}{a}$ and $\underset{\sim}{b}$.

ii. Find $\overrightarrow{OQ}$ in terms of $\mu, \underset{\sim}{a}$ and $\underset{\sim}{b}$.

iii. Hence show that $\lambda = \mu = \dfrac{2}{3}$.

b) Using vectors:

i. Prove that P, Q and B are collinear.

ii. Find $PQ : QB$

iii. Given $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$.

iv. Find the coordinates of the point Q.

v. Find the magnitude of $\angle AOB$ to the nearest tenth of a degree.

vi. Find the coordinates of the point R where R is on the line OA and OR is perpendicular to BR.

Question 9

$ABCD$ is a trapezium with the side AB parallel to the side DC as shown.
P and Q are the midpoints of sides AD and BC respectively.

a) Using $\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}$ to represent AB, AD and BC respectively express the vector $\overrightarrow{PQ}$ in two different ways as the sum of vectors along the sides of the trapezium.

b) By equating your two answers above, prove that the median, PQ is parallel to two sides of the trapezium.

c) Show that the length of the median, PQ, is equal to half the sum of the lengths if the parallel sides of a trapezium.

Question 10

Consider the two points, $A = (-2, 1, 2)$ and $B = (2, 3, 6)$.
Let $\underset{\sim}{a} = \overrightarrow{OA}$ and $\underset{\sim}{b} = \overrightarrow{OB}$.

a) Find the cosine of the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

Let C be the point on AB such that OC bisects $\angle AOB$.

b) Use an appropriate double angle formula, and your answer to **a**, find the cosine of $\angle AOC$.

c) Find $\hat{\underset{\sim}{a}}$ and $\hat{\underset{\sim}{b}}$.

Let X be the point that lies midway between the endpoints of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

d) Explain why X must lie on OC.

e) Find $\overrightarrow{OX}$.

f) If $\overrightarrow{AC} = \lambda\overrightarrow{AB}$ and $\overrightarrow{OX} = \beta\overrightarrow{OC}$, find λ and β.

g) Hence find the coordinates of C.

Question 11

Consider two points, $A = (-1, -2, 2)$ and $C = (4, 2, 4)$ relative to an origin O.

a) Show that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OC}$.

A third point exists $B = (x, y, z)$ such that $OABC$ forms a rectangle.

b) Show that $B = (3, 0, 6)$.

c) Prove that the diagonals of $OABC$ bisect one another.

Consider another point $V = (-1.5, 1.5, 3)$ such that $OABCV$ forms a rectangular based pyramid. Let E be the closest point to V on the plane $OABC$. Let M be the midpoint of AB.

d) If E is a point on OM, show that $E = (0.5, -0.5, 2)$

e) Hence, find the height of pyramid $OABCV$.

$\overrightarrow{OE}, \overrightarrow{AE}$ and $\overrightarrow{EV}$ are mutually perpendicular.
Vicky takes note of this and decides to reposition the origin to E so that the vertices of the pyramid can be described with simpler position vectors.
She does by defining new mutually perpendicular unit vectors relative to E as follows:

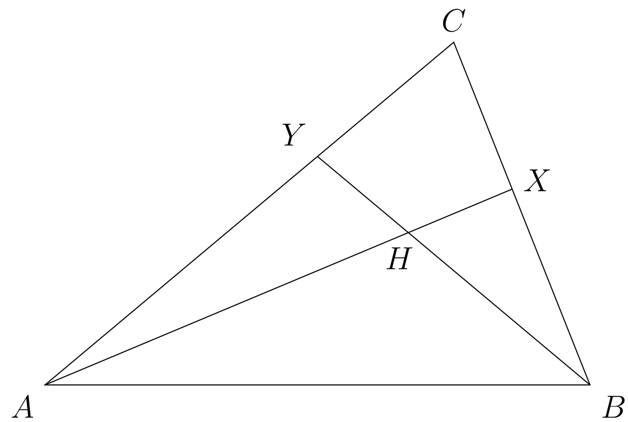
$$\begin{aligned} i^* &= \text{unit vector in the direction of } \overrightarrow{EM} \\ j^* &= \text{unit vector in the direction of } \overrightarrow{EA} \\ k^* &= \text{unit vector in the direction of } \overrightarrow{EV} \end{aligned}$$

f) Describe position vectors to O, A and V in terms of i^*, j^*, k^*

g) Describe position vectors to C in terms of i^*, j^*, k^*

Question 12

An altitude of a triangle is a line segment drawn from one vertex to the opposite side, meeting the opposite side at 90°. In the diagram on the left AX and BY are two altitudes of the triangle ABC.
H is the point of intersection AX and BY.
a) Express the vector $\overrightarrow{BC}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.



b) What is the scalar product $\overrightarrow{AH} \cdot \overrightarrow{BC}$?

c) What is the scalar product $\overrightarrow{AC} \cdot \overrightarrow{BH}$?

d) Express the vector $\overrightarrow{CH}$ in terms of $\overrightarrow{AC}$ and $\overrightarrow{AH}$.

e) Use a vector method (making use of all previous answers) to show that $\overrightarrow{CH} \cdot \overrightarrow{AB} = 0$

f) What is the geometric result you have proved in **part e.** ?