a) Let a regular n-gon be inscribed in a circle of radius r. Let A denote the area of the region inside the circle but not inside the n-gon. Prove that $A = \pi r^2 - \frac{nr^2}{2}\sin\left(\frac{2\pi}{n}\right)$.

b) Given that $\sin x \approx x \approx \tan x$ as $x \to 0^+$, prove that $\lim_{n\to\infty} A = 0$.

c) Let us consider another 'limiting polygon' case. Suppose we have a regular n-gon with fixed side lengths x. Find an expression for the area of this n-gon, denoted B. Show that $\lim_{n\to\infty} B = \infty$.

d) (d) illustrates the construction of what is known as an apeirogon**.** Is an apeirogon a circle? (Look up the definition of a circle.) If not, what is the difference between them?

e) Suppose that x is no longer fixed and allowed to depend on n. Suggest an formula for x, dependent on n such that $\lim_{n\to\infty} B$ is positive and finite, and prove that this is the case. Is this limiting shape a circle?